CS 4300: Compiler Theory

Chapter 6
Intermediate-Code Generation

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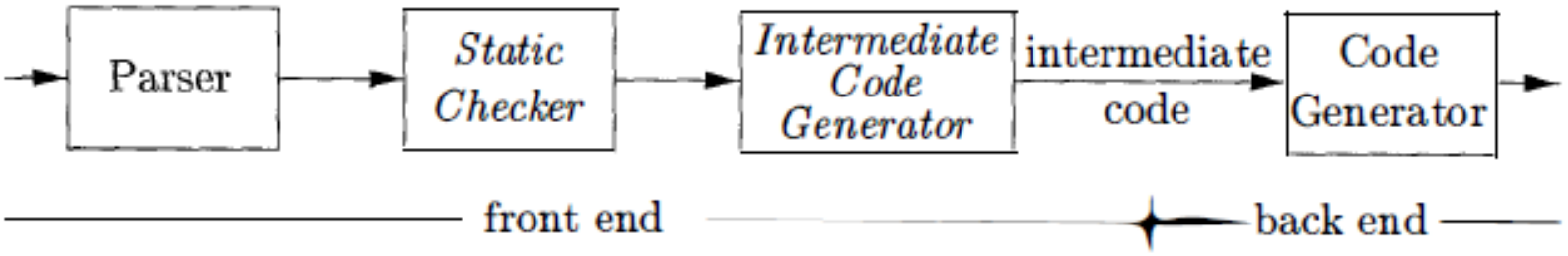
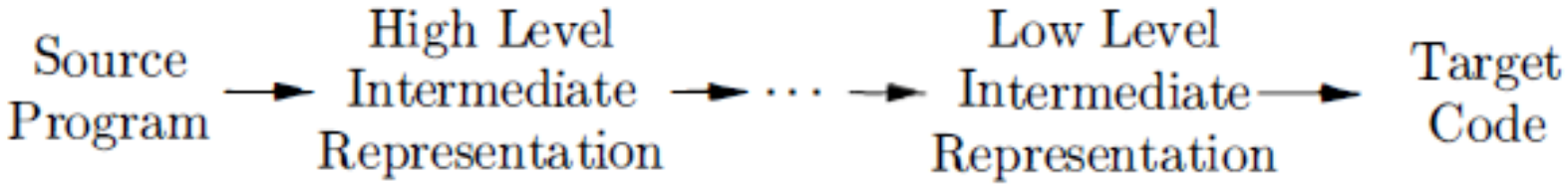
Introduction

Logical structure of a compiler front end



A sequence of intermediate representations



Syntax trees are high level

Three-address code can range from high-level to low-level, depending on the choice of operators

Static versus Dynamic Checking

- Static checking: checked at compile time
  - Compiler enforces programming language's static semantics
  - Typical examples of static checking:
    - Type checks
    - Flow-of-control checks
    - Uniqueness checks
    - Name-related checks
- Dynamic semantics: checked at run time
  - Compiler generates verification code to enforce programming language's dynamic semantics

Type Checking, Overloading, Coercion, Polymorphism

```
class X { virtual int m(); } *x;
class Y: public X { virtual int m(); } *y;
int op(int), op(float);
int f(float);
int a, c[10], d;

d = c + d;          // FAIL
*d = a;             // FAIL
a = op(d);          // OK: static overloading (C++)
a = f(d);           // OK: coersion of d to float
a = x->m();         // OK: dynamic binding (C++)
vector<int> v;      // OK: template instantiation
```

Flow-of-Control Checks

```
myfunc()
{ …
  break; // ERROR
}
```

```
myfunc()
{ …
  while (n)
  { …
    if (i>10)
      break; // OK
  }
}
```

```
myfunc()
{ …
  switch (a)
  { case 0:
      …
      break; // OK
    case 1:
      …
  }
}
```

Uniqueness Checks

```
myfunc()
{ int i, j, i; // ERROR
  …
}
```

```
cnufym(int a, int a) // ERROR
{   …
}
```

```
struct myrec
{ int name;
};
struct myrec // ERROR
{ int id;
};
```

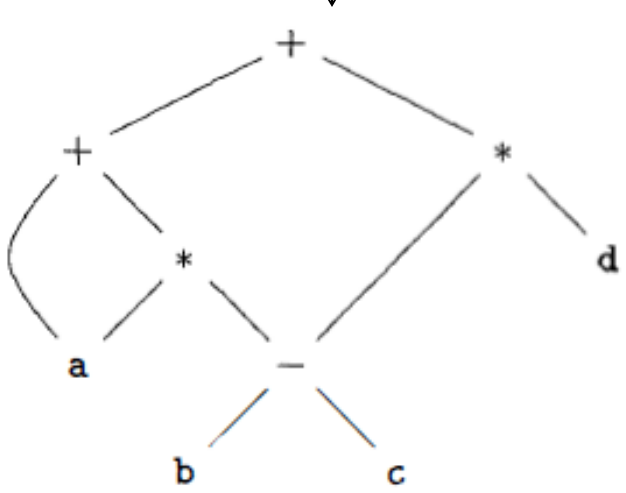
Outlines (Sections)

1. Variants of Syntax Trees
2. Three-Address Code
3. Types and Declarations
4. Translation of Expressions
5. Type Checking
6. Control Flow
7. Backpatching
8. Switch-Statements
9. Intermediate Code for Procedures

1. Variants of Syntax Trees

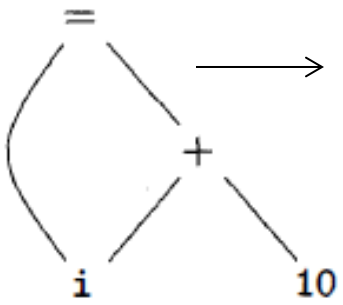
A directed acyclic graph (called a DAG) for an expression identifies the common subexpressions of the expression

a + a * (b - c) + (b - c) * d



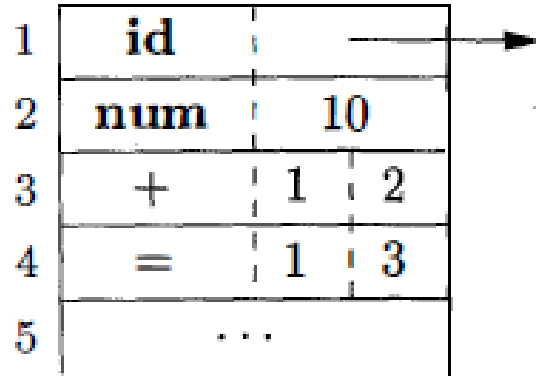
DAG

i = i + 10



DAG

to symbol table

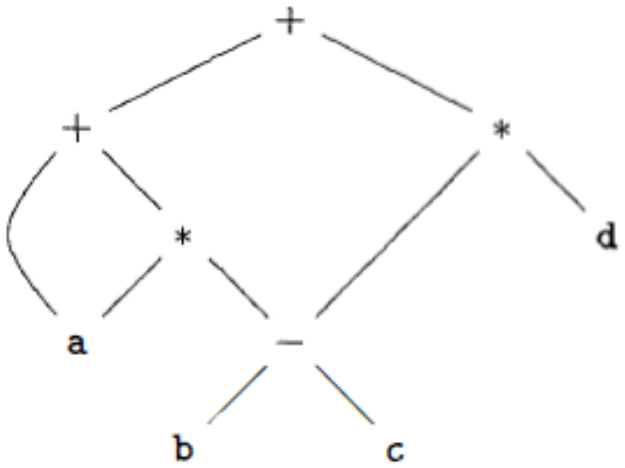


Array

Value number

2. Three-Address Code

In three-address code, there is at most one operator on the right side of an instruction. An address can be: name, constant, compiler-generated temporary.



DAG

$t_1 = b - c$
$t_2 = a * t_1$
$t_3 = a + t_2$
$t_4 = t_1 * d$
$t_5 = t_3 + t_4$

Three-address code

Common Three-Address Instructions

1. Assignment instruction — $x = y\ op\ z$
2. Assignment — $x = op\ y$
3. Copy instruction — $x = y$
4. Indexed copy instruction — $x = y[i]$ and $x[i] = y$
5. Address and pointer assignment: — $x = \&y$, $x = *y$, and $*x = y$
6. Unconditional jump — *goto L*
7. Conditional jump — *if x relop y goto L*
8. Conditional jump — *if x goto L* and *ifFalse x goto L*
9. Procedure call $p(x_1, x_2, \ldots, x_n)$: — *param* x_1
   param x_2
   ……
   param x_n
   call p, n

Two Ways of Assigning Labels to Three-Address Statements

```
do i = i+ l; while (a[i] < v) ;
```

```
L:   t1 = i + 1
     i = t1
     t2 = i * 8
     t3 = a [ t2 ]
     if t3 < v goto L
```

(a) Symbolic labels.

```
100: t1 = i + 1
101: i = t1
102: t2 = i * 8
103: t3 = a [ t2 ]
104: if t3 < v goto 100
```

(b) Position numbers.

Quadruples, Triples, and Indirect Triples

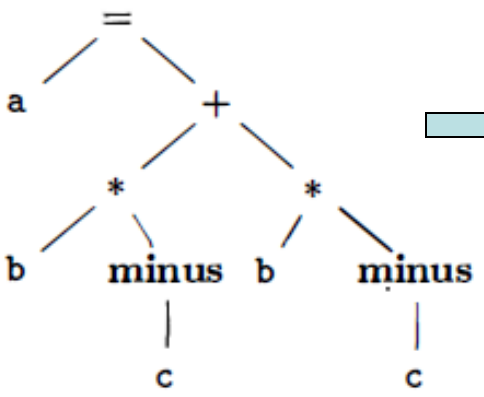
```
a = b*-c+b*-c;
```

```
t1 = minus c
t2 = b * t1
t3 = minus c
t4 = b * t3
t5 = t2 + t4
 a = t5
```

Three-address code

	op	arg_1	arg_2	result
0	minus	c		t_1
1	*	b	t_1	t_2
2	minus	c		t_3
3	*	b	t_3	t_4
4	+	t_2	t_4	t_5
5	=	t_5		a
	...			

Quadruples



Syntax tree

	op	arg_1	arg_2
0	minus	c	
1	*	b	(0)
2	minus	c	
3	*	b	(2)
4	+	(1)	(3)
5	=	a	(4)
	...		

Triples

	instruction
35	(0)
36	(1)
37	(2)
38	(3)
39	(4)
40	(5)
	...

+ Triples

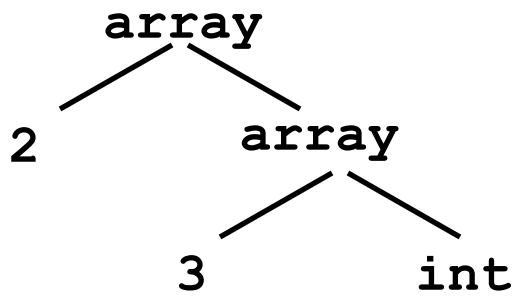
Indirect triples

3. Type Expressions

- A type expression is either a basic type or is formed by applying a type constructor to type expressions
  - Basic types: boolean, char, integer, float, etc.
  - Type constructors: pointer-to, array-of, records and classes, list-of, templates, and functions ($s \rightarrow t$).
  - Type names: typedefs in C and named types in Pascal
- Type expressions may contain variables whose values are type expressions

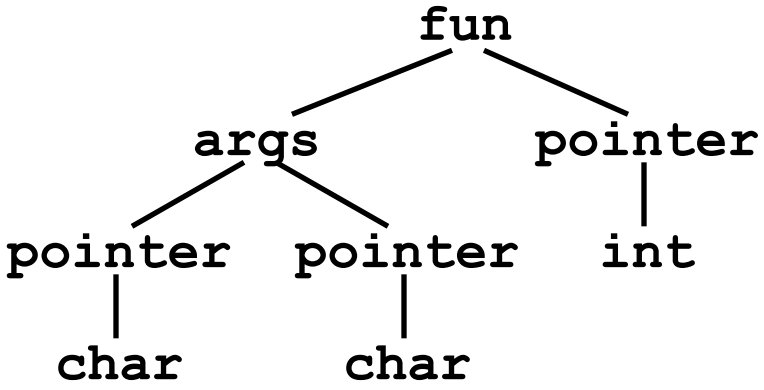
Graph Representations for Type Expressions

```
int [2][3]
```

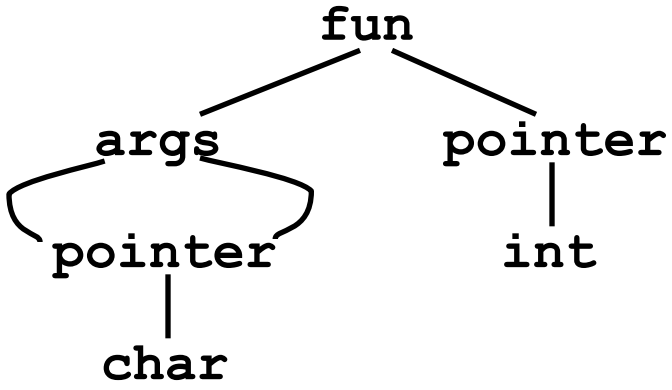


Tree

```
int *fun(char*,char*)
```



Tree



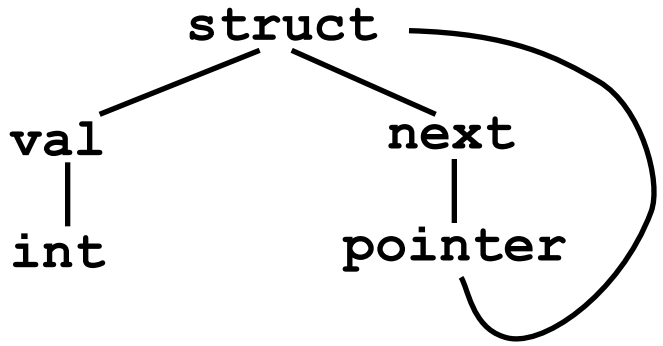
DAG

Cyclic Graph Representations

Source program

```
struct Node
{ int val;
  struct Node *next;
};
```

Internal compiler representation of the **`Node`** type: cyclic graph

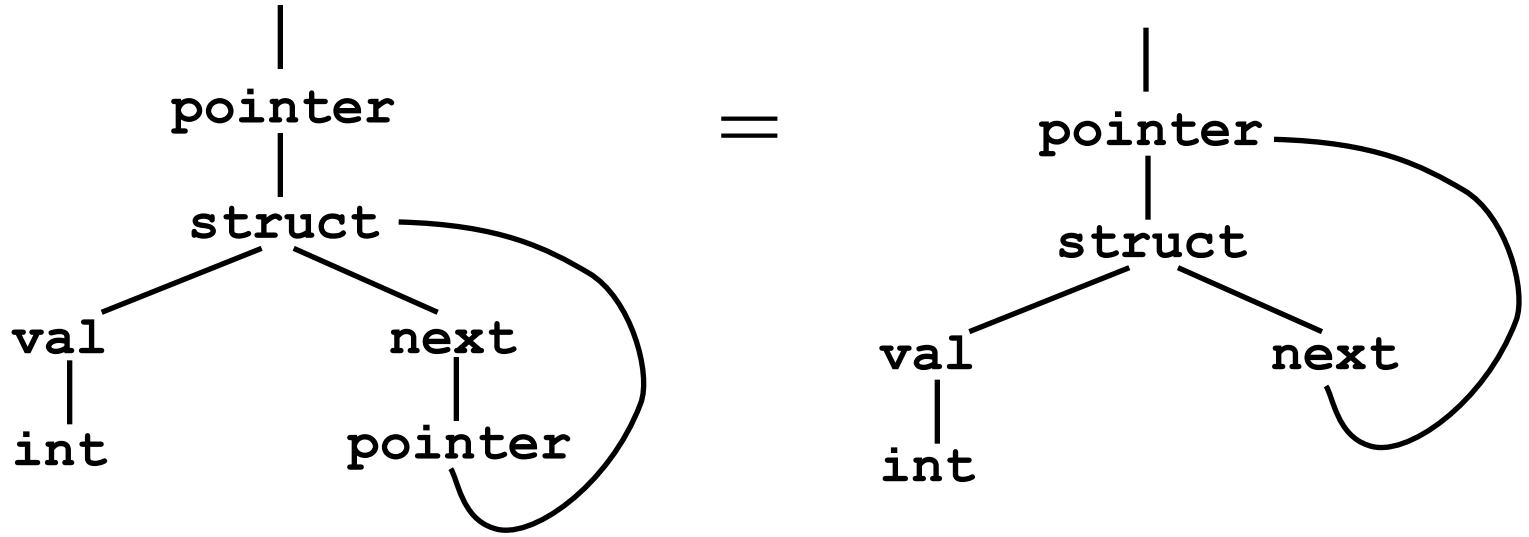


Type Equivalence

- When type expressions are represented by graphs, two types are structurally equivalent if and only if one of the following conditions is true:
  - They are the same basic type.
  - They are formed by applying the same constructor to structurally equivalent types .
  - One is a type name that denotes the other .
- If type names are treated as standing for themselves, then the first two conditions in the above definition lead to name equivalence of type expressions

Structural Equivalence Example

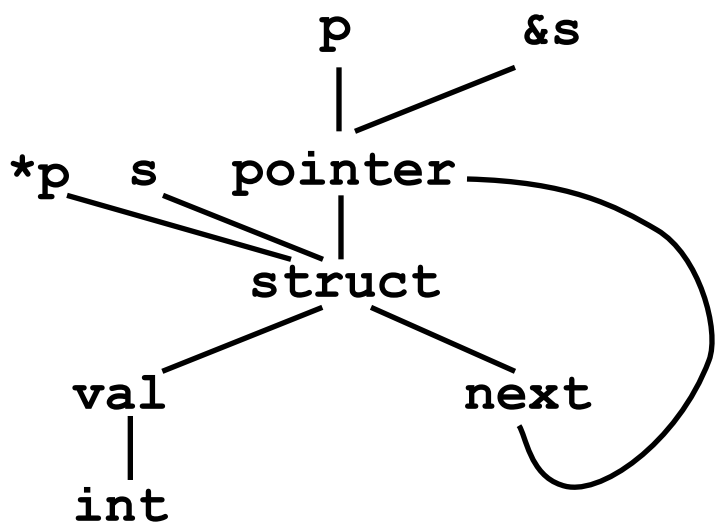
- Two types are the same if they are *structurally identical*
- Used in C/C++, Java, C#



Type Equivalence Examples

```
struct Node
{ int val;
  struct Node *next;
};

struct Node s, *p;
p = &s; // OK
*p = s; // OK
p = s;  // ERROR
```

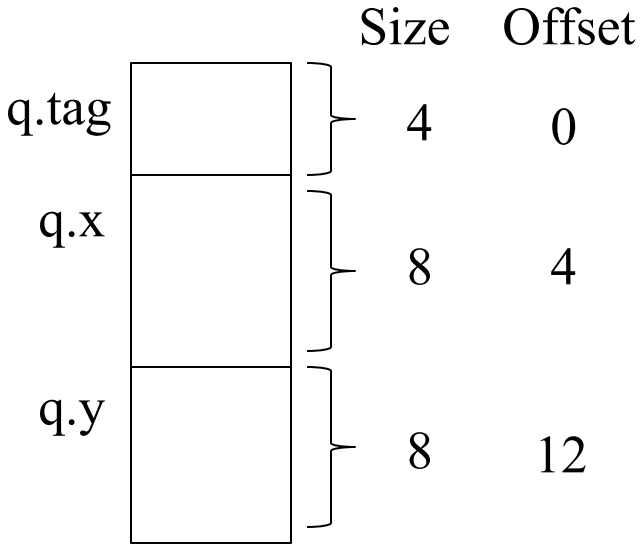


Storage Layout for Local Names

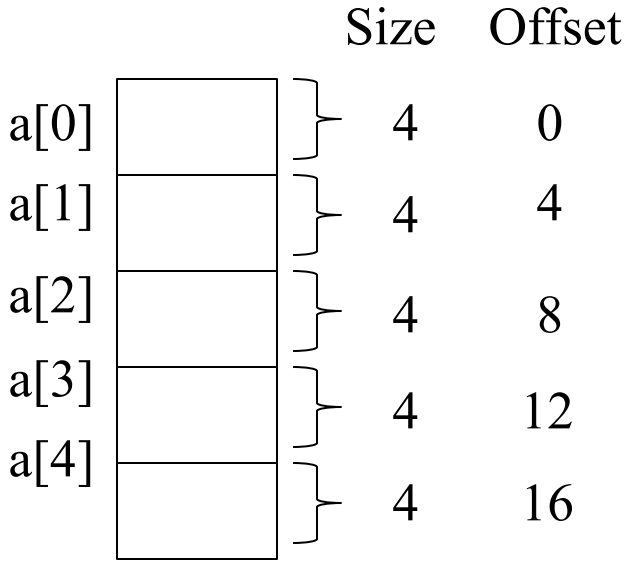
Type Declarations

$$
\begin{array}{lcl}
D & \rightarrow & T\ \mathbf{id}\ ;\ D \mid \epsilon \\
T & \rightarrow & B\ C \mid \mathbf{record}\ '\{'\ D\ '\}' \\
B & \rightarrow & \mathbf{int} \mid \mathbf{float} \\
C & \rightarrow & \epsilon \mid [\ \mathbf{num}\ ]\ C
\end{array}
$$

record { int tag; float x; float y; } q;



int [5] a;



Computing Types and Their Widths

Type Declarations

$$
\begin{array}{lcl}
D & \rightarrow & T\ \mathbf{id}\ ;\ D \mid \epsilon \\
T & \rightarrow & B\ C \mid \mathbf{record}\ '\{'\ D\ '\}' \\
B & \rightarrow & \mathbf{int} \mid \mathbf{float} \\
C & \rightarrow & \epsilon \mid [\ \mathbf{num}\ ]\ C
\end{array}
$$

$$
\begin{array}{lcll}
T & \rightarrow & B & \{\ t = B.type;\ w = B.width;\ \} \\
 & & C & \{\ T.type = C.type; T.width = C.width;\ \} \\
B & \rightarrow & \mathbf{int} & \{\ B.type = integer;\ B.width = 4;\ \} \\
B & \rightarrow & \mathbf{float} & \{\ B.type = float;\ B.width = 8;\ \} \\
C & \rightarrow & \epsilon & \{\ C.type = t;\ C.width = w;\ \} \\
C & \rightarrow & [\ \mathbf{num}\ ]\ C_1 & \{\ array(\mathbf{num}.value,\ C_1.type); \\
 & & & \quad C.width = \mathbf{num}.value \times C_1.width;\ \}
\end{array}
$$

Sequences of Declarations

Computing the relative addresses of declared names

$P \rightarrow$ $\{ top = new\ Evn();\ offset = 0; \}$
D

$D \rightarrow T\ \mathbf{id}\ ;$ $\{ top.put(\mathbf{id}.lexeme,\ T.type,\ offset);$
$offset = offset + T.width; \}$
D_1

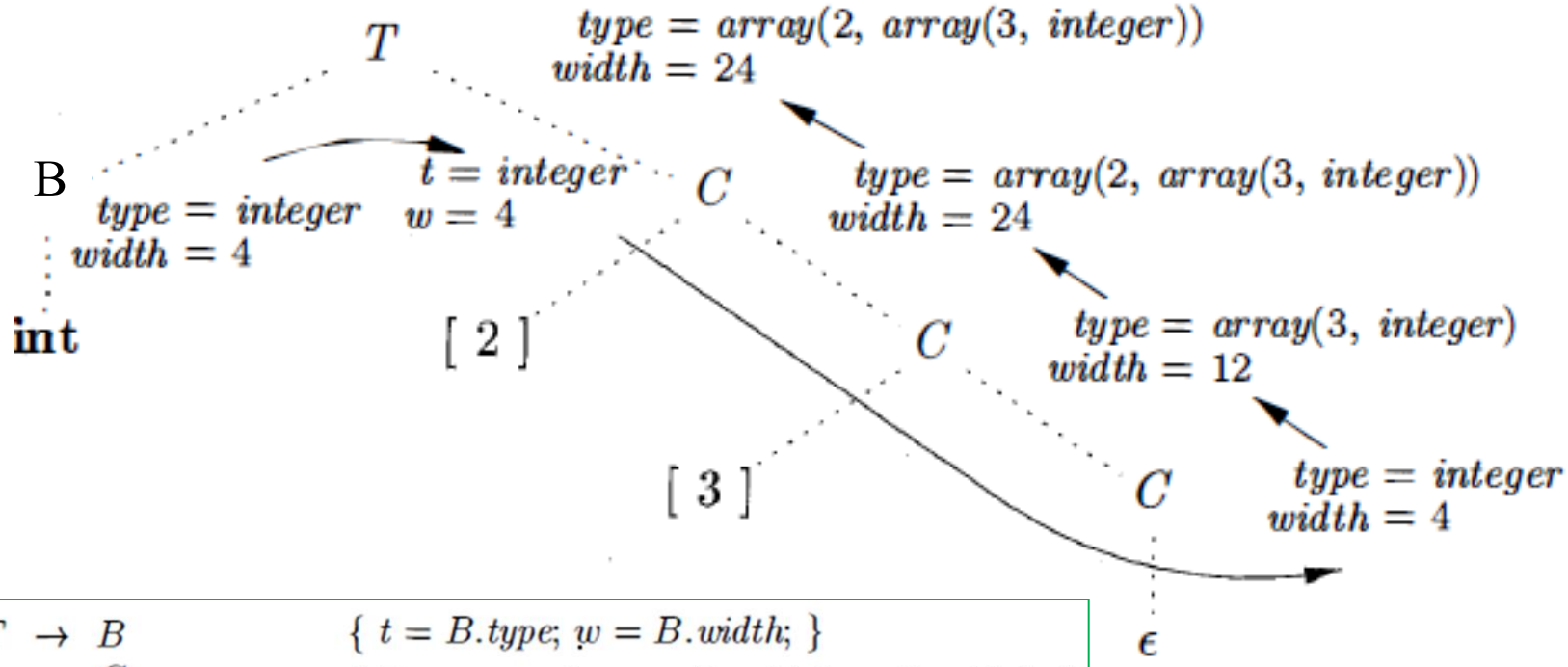
$D \rightarrow \epsilon$

Handling of field names in records

$T \rightarrow \mathbf{record}\ '\{'$ $\{ Env.push(top);\ top = \mathbf{new}\ Env();$
$Stack.push(offset);\ offset = 0; \}$

$D\ '\}'$ $\{ T.type = record(top);\ T.width = offset;$
$top = Env.pop();\ offset = Stack.pop(); \}$

Example:
Annotated Parse Tree for int [2][3]



$T \rightarrow B$		$\{ t = B.type;\ w = B.width; \}$
	C	$\{ T.type = C.type; T.width = C.width; \}$
$B \rightarrow$	**int**	$\{ B.type = integer;\ B.width = 4; \}$
$C \rightarrow$	ϵ	$\{ C.type = t;\ C.width = w; \}$
$C \rightarrow$	[**num**] C_1	$\{ array(\mathbf{num}.value,\ C_1.type);$ $C.width = \mathbf{num}.value \times C_1.width; \}$

Example:
Determine types and relative addresses

float x;
record { float x; float y; } p;
record { int tag; float x; float y; } q;

$P \rightarrow$	D	$\{ offset = 0; \}$
$D \rightarrow$	T **id** ; D_1	$\{ top.put(\textbf{id}.lexeme, T.type, offset);$ $offset = offset + T.width; \}$
$D \rightarrow$	ϵ	

$T \rightarrow$	**int**	$\{ T.type = integer; T.width = 4; \}$
$T \rightarrow$	**float**	$\{ T.type = float; T.width = 8; \}$

$T \rightarrow$	**record** '{'	$\{ Env.push(top); top = \textbf{new}\ Env();$ $Stack.push(offset); offset = 0; \}$
	D '}'	$\{ T.type = record(top); T.width = offset;$ $top = Env.pop(); offset = Stack.pop(); \}$

Line, id, type, offset, width

4. Translation of Expressions

Example
a = b + - c

PRODUCTION	SEMANTIC RULES
$S \rightarrow id = E\ ;$	S.code = E.code || gen(top.get(id.lexeme) '=' E.addr)
$E \rightarrow E_1 + E_2$	E.addr = **new** Temp() E.code = E_1.code || E_2.code || gen(E.addr '=' E_1.addr '+' E_2.addr)
| $- E_1$	E.addr = **new** Temp () E.code = E_1.code || gen(E.addr '=' 'minus' E_1.addr)
| (E_1)	E.addr = E_1.addr E.code = E_1.code
| **id**	E.addr = top.get(id.lexeme) E.code = ' '

t_1 = **minus** c
t_2 = b + t_1
a = t_2

Figure 6.19: Three-address code for expressions

Translation of Expressions (cont.)
Incremental Translation

$S \rightarrow id = E$;	{ gen(top.get(id.lexeme) '=' E.addr) ; }
$E \rightarrow E_1 + E_2$	{ E.addr = new Temp(); gen(E.addr '=' E_1.addr '+' E_2.addr) ; }
| - E_1	{ E.addr = new Temp() ; gen(E.addr '=' 'minus' E_1.addr) ; }
| (E_1)	{ E.addr = E_1.addr; }
| **id**	{ E.addr = top.get(id.lexeme) ;}

Figure 6.20: Generating three-address code for expressions incrementally

In the incremental approach, gen not only constructs a three-address instruction, but also appends the instruction to the sequence of instructions generated so far.

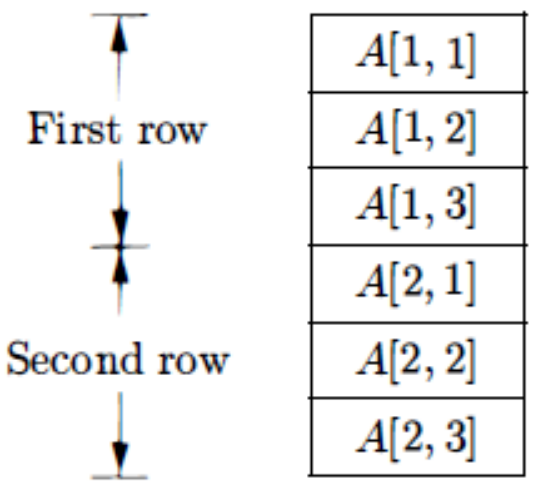
Addressing Array Elements

$$a[i].addr = base + i \times w$$

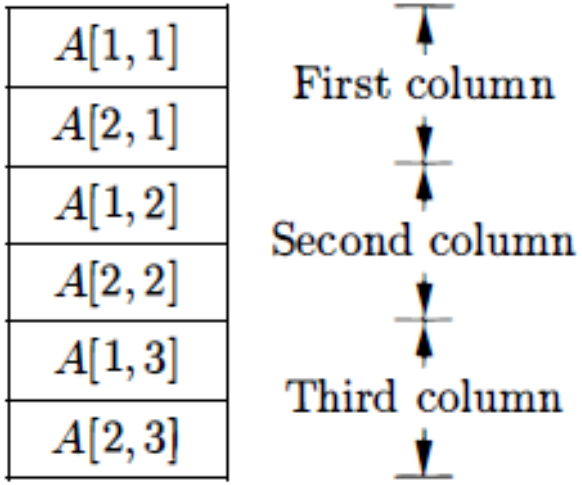
$$A[i_1][i_2].addr = base + i_1 \times w_1 + i_2 \times w_2$$

$$A[i_1][i_2]\ldots[i_k].addr = base + i_1 \times w_1 + i_2 \times w_{2.} \ldots + i_k \times w_k \qquad (6.4)$$

Layouts for a two-dimensional array



(a) Row Major



(b) Column Major

Translation of Array References

Figure 6.22: Semantic actions for array references

$S \rightarrow$	**id** = E ;	{ $gen(\ top.get(\textbf{id}.lexeme)\ '='\ E.addr);$ }
|	L = E ;	{ $gen(L.addr.base\ '['\ L.addr\ ']'\ '='\ E.addr);$ }
$E \rightarrow$	E_1 + E_2	{ $E.addr = \textbf{new}\ Temp\,();$ $gen(E.addr\ '='\ E_1.addr\ '+'\ E_2.addr);$ }
|	**id**	{ $E.addr = top.get(\textbf{id}.lexeme);$ }
|	L	{ $E.addr = \textbf{new}\ Temp\,();$ $gen(E.addr\ '='\ L.array.base\ '['\ L.addr\ ']');$ }
$L \rightarrow$	**id** [E]	{ $L.array = top.get(\textbf{id}.lexeme);$ $L.type = L.array.type.elem;$ $L.addr = \textbf{new}\ Temp\,();$ $gen(L.addr\ '='\ E.addr\ '*'\ L.type.width);$ }
|	L_1 [E]	{ $L.array = L_1.array;$ $L.type = L_1.type.elem;$ $t = \textbf{new}\ Temp\,();$ $L.addr = \textbf{new}\ Temp\,();$ $gen(t\ '='\ E.addr\ '*'\ L.type.width);$ } $gen(L.addr\ '='\ L_1.addr\ '+'\ t);$ }

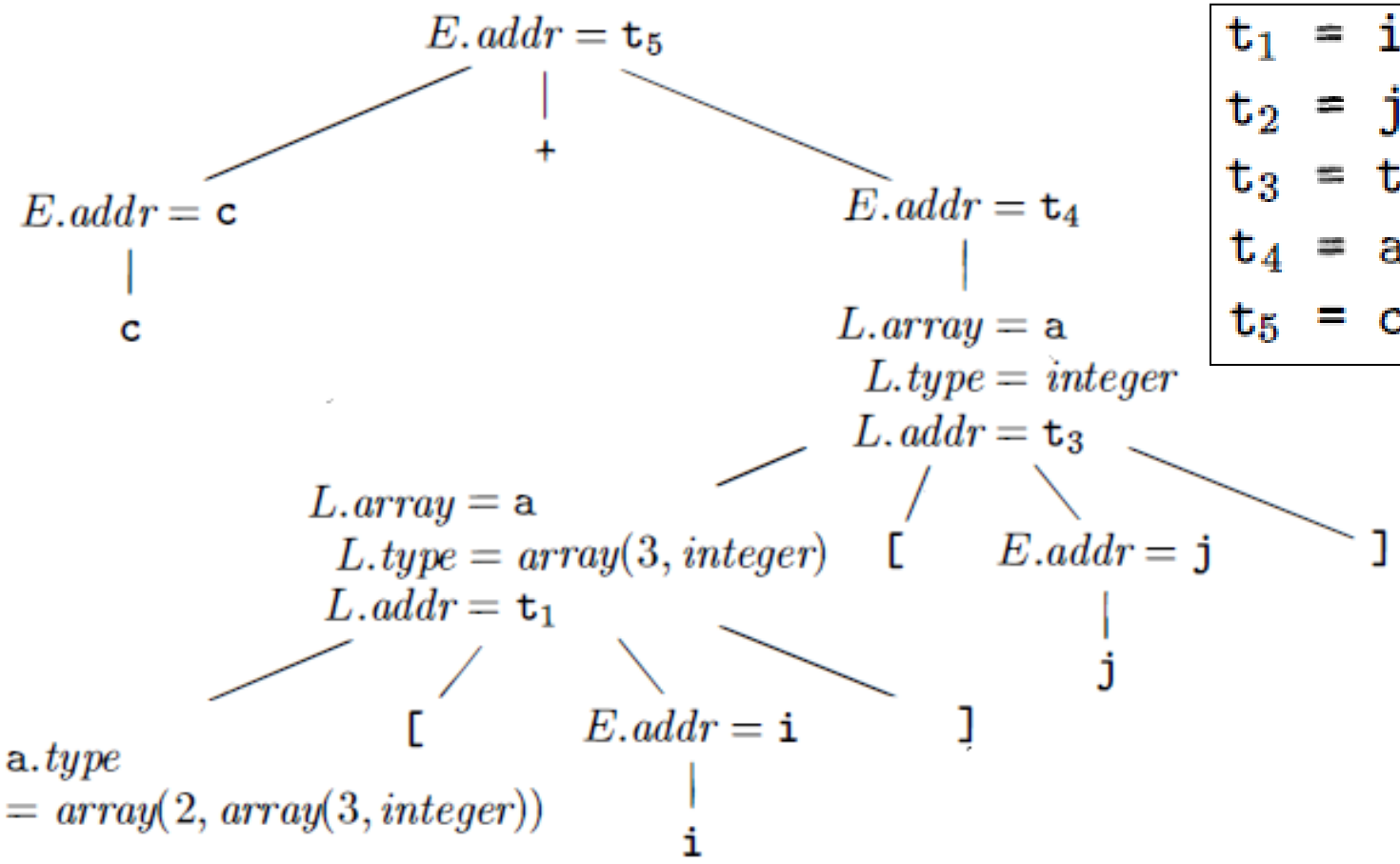
Translation of Array References (Cont.)

- Nonterminal L has three synthesized attributes:
  1. L.addr denotes a temporary that is used while computing the offset for the array reference by summing the $\mathbf{i}_j \times \mathbf{w}_j$ in (6.4)
  2. L.array is a pointer to the symbol-table entry for the array name.
     - L.array.base is the base address of the array.
     - L.array.type is the type of the array.
  3. L.type is the type of the subarray generated by L.
- Assume t is a type, then
  - t.width represents the width.
  - t.elem gives the element type.

Example 6.12

a is a 2×3 array of integers
i, **j**, and **c** are integers

Annotated parse tree for c + a[i][j]



Three-address code for c + a[i][j]

```
t1 = i * 12
t2 = j * 4
t3 = t1 + t2
t4 = a [ t3 ]
t5 = c + t4
```

5. Type Checking

- To do type checking a compiler needs to assign a type expression to each component of the source program.
- The compiler must then determine that these type expressions conform to a collection of logical rules that is called the type system for the source language
- Type checking can take on two forms:
  - Synthesis
  - Inference

Rules for Type Checking

- Type synthesis builds up the type of an expression from the types of its subexpressions.
- It requires names to be declared before they are used.

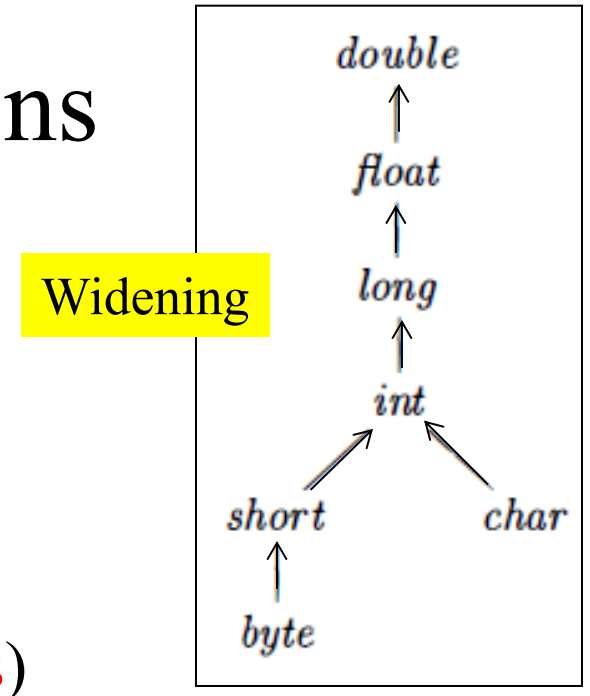
> **if** f has type s → t **and** x has type s,
> **then** expression f (x) has type t

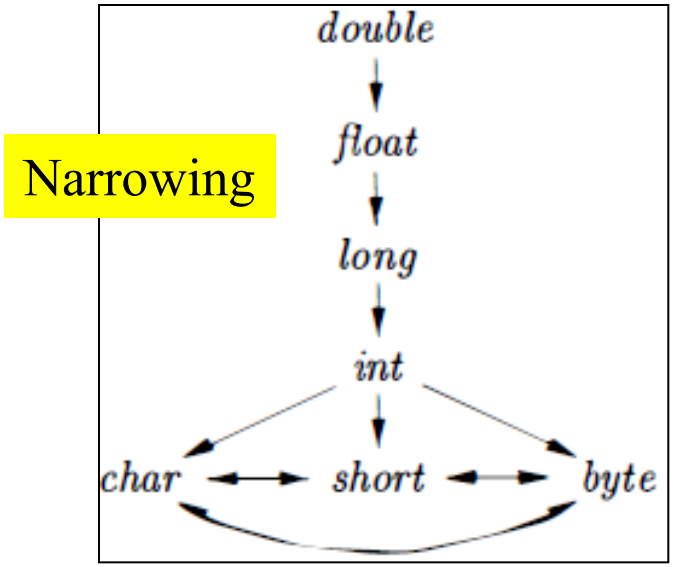
- Type inference determines the type of a language construct from the way it is used.
- It does not require names to be declared

> **if** f (x) is an expression,
> **then** for some α and β, f has type α → β and x has type α

Type Conversions

- Widening conversions
  - preserve information
- Narrowing conversions
  - lose information
- Coercions (implicit conversions)
  - are done automatically by the compiler.
- Casts (explicit conversions)
  - are done by programmer to write something to cause the conversion.





Introducing Type Conversions into Expression Evaluation

$$
\begin{aligned}
E \rightarrow E_1 + E_2 \quad \{\ & E.type = max(E_1.type, E_2.type); \\
& a_1 = widen(E_1.addr, E_1.type, E.type); \\
& a_2 = widen(E_2.addr, E_2.type, E.type); \\
& E.addr = \textbf{new}\ Temp\,(); \\
& gen(E.addr\ '='\ a_1\ '+'\ a_2);\ \}
\end{aligned}
$$

max(t_1, t_2) returns the maximum (or least upper bound) of the two types t_1 and t_2 in the widening hierarchy.

widen(a, t, w) generates type conversions if needed to widen an address a of type t into a value of type w.

x = 2 + 3.14

t_1 = (**float**) 2
t_2 = t_1 + 3.14
x = t_2

Overloading of Functions and Operators

Overloaded function examples

```
void err () { ... }
void err (String s) { … }
```

A type-synthesis rule for overloaded functions

if f can have type $s_i \rightarrow t_i$, for $1 \leq i \leq n$, where $s_i \neq s_j$ for $i \neq j$
and x has type s_k, for some $1 \leq k \leq n$
then expression $f(x)$ has type t_k

6.5.4 Type Inference and Polymorphic Functions

The term "polymorphic" refers to any code fragment that can be executed with arguments of different types

ML program for the length of a list

```
fun length(x) =
if null(x) then 0 else length(tl(x)) + 1;
```

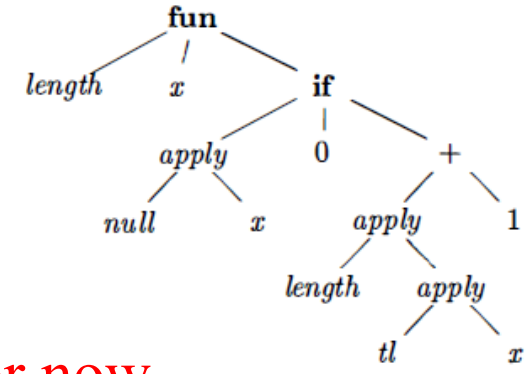
The type of *length*

$\forall\alpha\ list(\alpha) \rightarrow integer$

Example of use of *length*

length([“sun”, “mon”, “tue”]) +
length([10, 9, 8, 7]) returns 7

Abstract syntax tree



Note: 6.5.4 and 6.5.5 need to be skipped for now.

Substitutions, Instances, and Unification

- A substitution S is a mapping from type variables to type expressions.
  - $S(t)$ = the result of applying the substitution S to the variables in type expression t.
    - $S(\alpha) = integer$
    - $t = list(\alpha)$, then $S(t) = list(integer)$
    - $t = \alpha \rightarrow \alpha$, then $S(t) = integer \rightarrow integer$
- $S(t)$ is called an instance of t.
- A substitution S is a *unifier* of two types t_1 and t_2 (t_1 and t_2 unify), if $S(t_1) = S(t_2)$.
- In the type inference algorithm, we *substitute* type variables by types to create type *instances*

Inferring a type for the function *length*

fun *length*(*x*) = **if** *null*(*x*) then 0 **else** *length*(*tl*(*x*)) + 1;

LINE	EXPRESSION : TYPE	UNIFY
1)	$length : \beta \rightarrow \gamma$	
2)	$x : \beta$	
3)	$\mathbf{if} : boolean \times \alpha_i \times \alpha_i \rightarrow \alpha_i$	
4)	$null : list(\alpha_n) \rightarrow boolean$	
5)	$null(x) : boolean$	$list(\alpha_n) = \beta$
6)	$0 : integer$	$\alpha_i = integer$
7)	$+ : integer \times integer \rightarrow integer$	
8)	$tl : list(\alpha_t) \rightarrow list(\alpha_t)$	
9)	$tl(x) : list(\alpha_t)$	$list(\alpha_t) = list(\alpha_n)$
10)	$length(tl(x)) : \gamma$	$\gamma = integer$
11)	$1 : integer$	
12)	$length(tl(x)) + 1 : integer$	
13)	$\mathbf{if}(\cdots) : integer$	

$\forall \alpha_n.\ list(\alpha_n) \rightarrow integer$

6.5.5: An Algorithm for Unification

Examples 6.18: Consider the two type Expressions t_1, t_2, and the substitution S

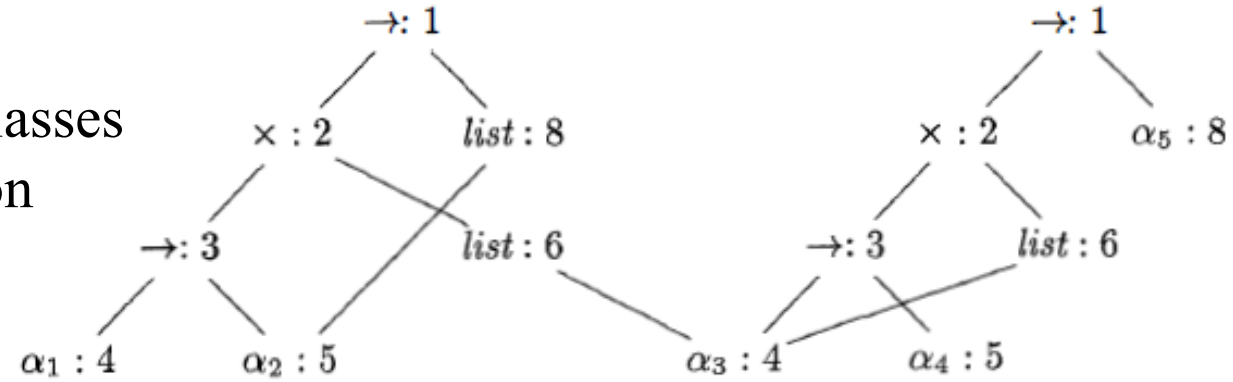
$t_1 = ((\alpha_1 \rightarrow \alpha_2) \times list(\alpha_3)) \rightarrow list(\alpha_2)$

$t_2 = ((\alpha_3 \rightarrow \alpha_4) \times list(\alpha_3)) \rightarrow \alpha_5$

x	$S(x)$
α_1	α_1
α_2	α_2
α_3	α_1
α_4	α_2
α_5	$list(\alpha_2)$

$S(t_1) = S(t_2) = ((\alpha_1 \rightarrow \alpha_2) \times list(\alpha_1)) \rightarrow list(\alpha_2)$

Equivalence classes after unification



An Algorithm for Unification(Cont.)

```
boolean unify(Node m, Node n) {
        s = find(m); t = find(n);
        if ( s = t ) return true;
        else if ( nodes s and t represent the same basic type ) return true;
        else if (s is an op-node with children s1 and s2 and
                        t is an op-node with children t1 and t2) {
                union(s, t);
                return unify(s1, t1) and unify(s2, t2);
        }
        else if s or t represents a variable {
                union(s, t);
                return true;
        }
        else return false;
}
```