CS 4410

Automata, Computability, and Formal Language

Dr. Xuejun Liang

Regular Grammar

Definition 3.3: A grammar G=(V, T, S, P) is said to be right-linear if all productions are of the form

$$A \rightarrow xB$$
$$A \rightarrow x$$

Where A, B $\in$ V, and x $\in$ T*. A grammar is said to be left-linear if all productions are of the form

$$A \rightarrow Bx$$
$$A \rightarrow x$$

A regular grammar is one that is either right-linear or left-linear.

A linear grammar is a grammar in which at most one variable can occur on the right side of any production.

Examples

Example 3.13: $G_1=(\{S\}, \{a, b\}, S, P_1)$, with P_1 given as

$$S \rightarrow abS \mid a.$$

is right-linear. $L(G_1) = ?$

$G_2=(\{S, S_1, S_2\}, \{a, b\}, S, P_2)$, with P_2 given as

$$S \rightarrow S_1ab,\ S_1 \rightarrow S_1ab \mid S_2,\ S_2 \rightarrow a.$$

is left-linear. $L(G_2)=?$

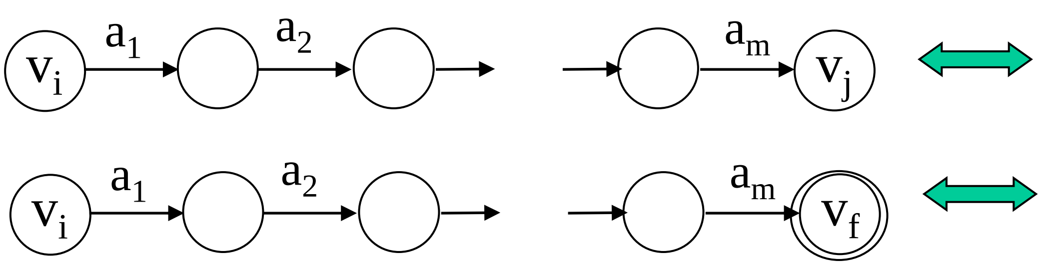
Example 3.14: $G=(\{S, A, B\}, \{a, b\}, S, P)$, with P given as

$$S \rightarrow A,\ A \rightarrow aB \mid \lambda,\ B \rightarrow Ab$$

is not regular. But, it a linear grammar.

Right-Linear Grammars Generate Regular Languages

Theorem 3.3: Let G=(V, T, S, P) be a right-linear grammar. Then L(G) is a regular language.



$V_i \rightarrow a_1 a_2 \ldots a_m V_j$

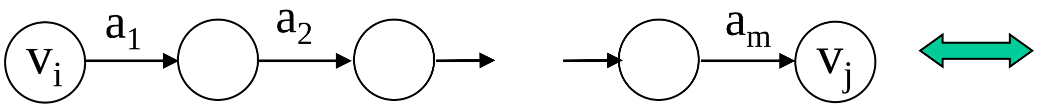
$V_i \rightarrow a_1 a_2 \ldots a_m$

Example 3.15: Construct a finite automaton that accepts the language generated by the grammar

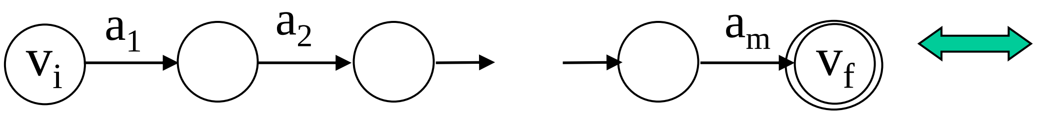
$$V_0 \rightarrow aV_1$$
$$V_1 \rightarrow abV_0 \mid b$$

Right-Linear Grammars for Regular Languages

Theorem 3.4: If L is a regular language on alphabet Σ. Then there exists a right-linear grammar G=(V, T, S, P) such that L=L(G).

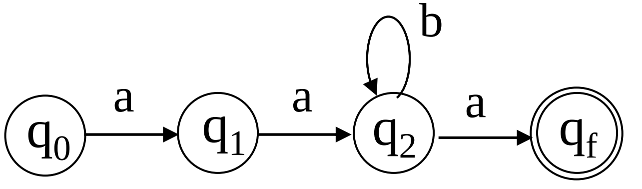


$V_i \rightarrow a_1a_2\ldots a_mV_j$



$V_i \rightarrow a_1a_2\ldots a_m$

Example 3.16: Construct a right-linear grammar for L(aab*a).



Equivalence of Regular Grammar and Regular Language as well as Regular Expression, NFA, or DFA