CS 4410

Automata, Computability, and Formal Language

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Regular Expressions Denote Regular Languages

Theorem 3.1: Let r be a regular expression. Then there exists some nondeterministic finite accepter that accepts L(r). Consequently, L(r) is a regular language.

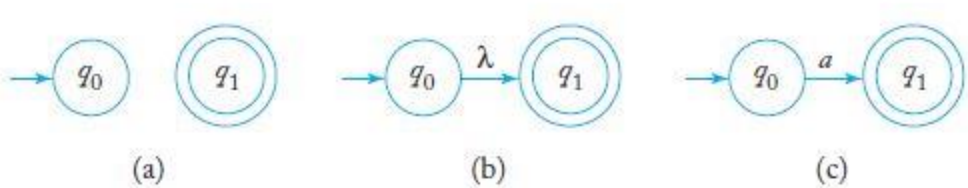


FIGURE 3.1 (a) nfa accepts $\varnothing$. (b) nfa accepts $\{\lambda\}$. (c) nfa accepts $\{a\}$.

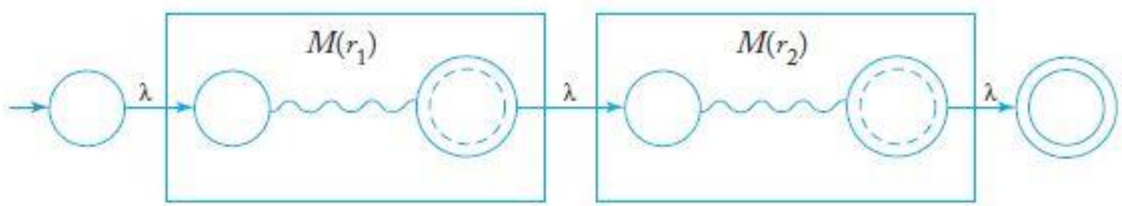


FIGURE 3.4 Automaton for $L(r_1 r_2)$.

Regular Expressions Denote Regular Languages

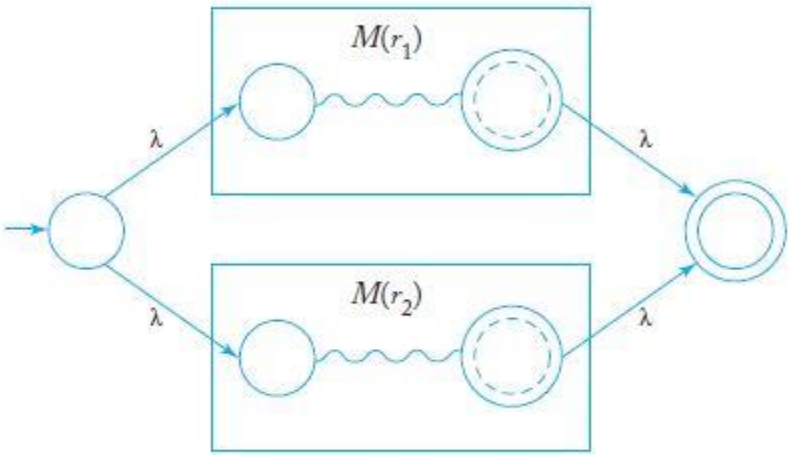


FIGURE 3.3 Automaton for $L(r_1 + r_2)$.

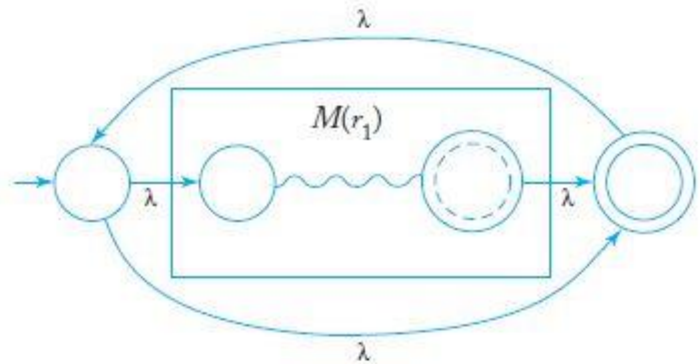


FIGURE 3.5 Automaton for $L(r_1^*)$.

NFA or DFA M ⟺ Regular language L(M)

? ⇓ ⇑ Theorem 3.1

Regular expression r ⇄ Regular language L(r)

?

Regular Expressions Denote Regular Languages

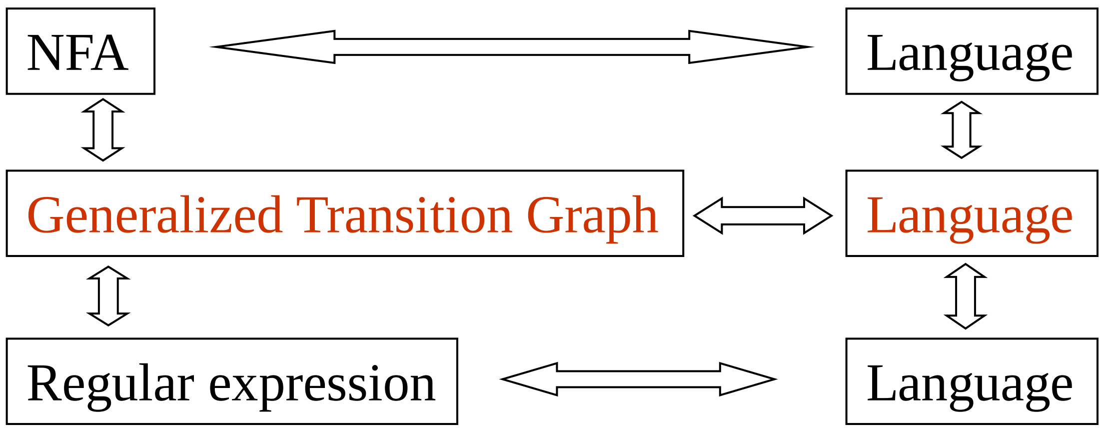
Example 3.7

Find an nfa which accepts L(r), where

$r = (a + bb)^* (ba^* + \lambda)$

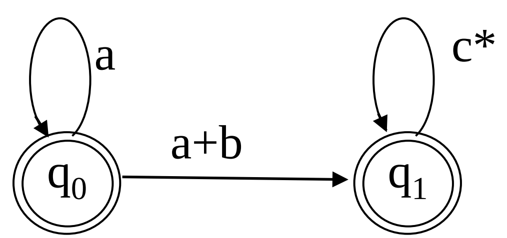
Generalized Transition Graph

In generalized transition graph, edges are regular expressions



Example 3.8

Find the language accepted by the generalized transition graph

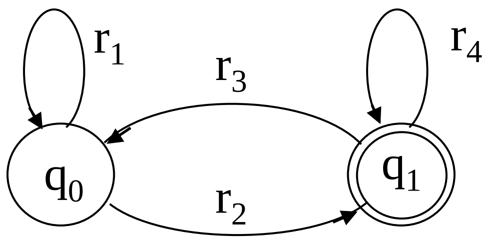


Regular Expressions for Regular Languages

Theorem 3.2: Let L be a regular language. Then there exists a regular expression r such that $L(r) = L$.

Proof Ideals

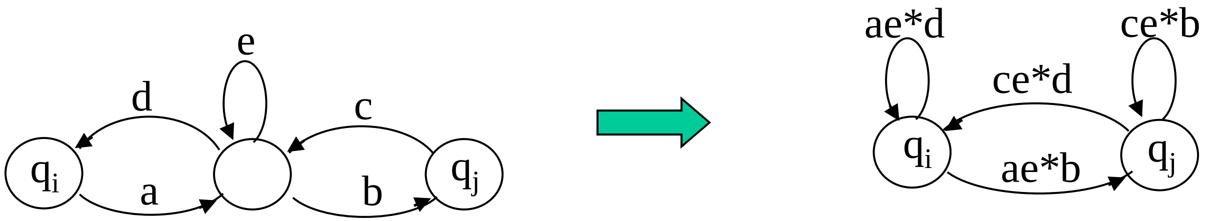
1. Let an NFA M accept L. Assume M has only one final state that is different with the initial state.
2. Convert M to an equivalent generalized transition graph by removing all states except the initial state and the final state.



3. The regular expression is

$$r = r_1{}^*r_2(r_4+r_3r_1{}^*r_2)^*$$

Transition Graph → Generalized Transition Graph

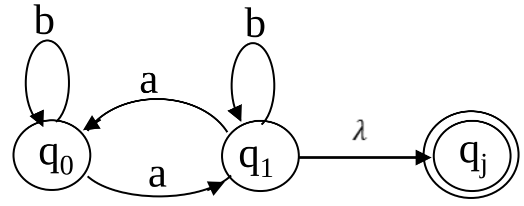


Transition Graph

Generalized Transition Graph

Example 3.9:

Convert the nfa to generalized transition graph

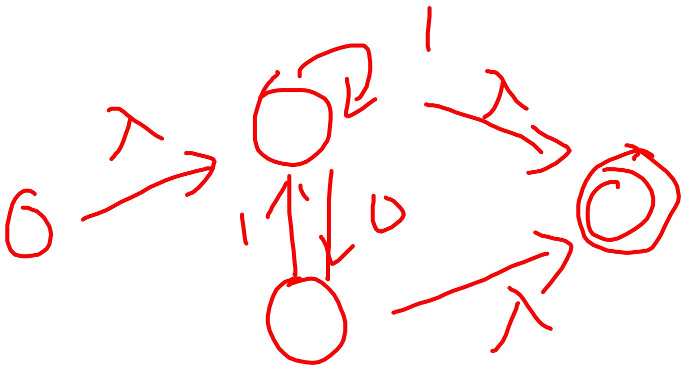


Example: Find a regular expression for the language

$L(r) = \{ w \in \{0,1\}^*$: w has no pair of consecutive zeros$\}$

Example: Find a regular expression for the language

L(r) = { w∈{0,1}*: w has no pair of consecutive zeros}



Regular Grammar

Definition 3.3: A grammar G=(V, T, S, P) is said to be right-linear if all productions are of the form

$$A \rightarrow xB$$
$$A \rightarrow x$$

Where A, B $\in$ V, and x $\in$ T*. A grammar is said to be left-linear if all productions are of the form

$$A \rightarrow Bx$$
$$A \rightarrow x$$

A regular grammar is one that is either right-linear or left-linear.

A linear grammar is a grammar in which at most one variable can occur on the right side of any production.