Coding and Information Theory Overview
Chapter 1: Source Coding

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The third Lecture of Chapter 1

Quick Review of Last Lecture

- Uniquely Decodable Code
  - Compute C_n and C_∞
  - Compute counterexample to show a code C is not u.d.
  - Proof of the Sardinas-Patterson Theorem (Theorem 1.10)
- Instantaneous Code
  - Prefix Code
  - Constructing Instantaneous Code
  - Why is there a prefix code for a given set of word lengths or not?

1.5 Kraft’s Inequality

- Theorem 1.20
  - There is an instantaneous r-ary code C with word-lengths $l_1, \dots, l_q$, if and only if

$$\sum_{i=1}^{q} \frac{1}{r^{l_i}} \leq 1\,. \qquad (1.5)$$

1.6 McMillan's Inequality

- Theorem 1.21
  - There is a uniquely decodable r-ary code C with word-lengths $l_1, \ldots, l_q$, if and only if

$$\sum_{i=1}^{q} \frac{1}{r^{l_i}} \leq 1 . \qquad (1.6)$$

- Corollary 1.22
  - There is an instantaneous r-ary code with word-lengths $l_1, \ldots, l_q$, if and only if there is a uniquely decodable r-ary code with these word-lengths .

1.7 Comments on Kraft's and McMillan's Inequalities

- Comment 1.23
  - Theorems 1.20 and 1.21 do not say that an r-ary code with word-lengths $l_1, \ldots, l_q$ is instantaneous or uniquely decodable if and only if $\sum r^{-l_i} \leq 1$
  - Examples: $C = \{0, 01, 011\}$ and $C = \{0, 01, 001\}$
- Comment 1.24
  - Theorems 1.20 and 1.21 assert that if $\sum r^{-l_i} \leq 1$ then there exist codes with these parameters which are instantaneous and uniquely decodable.
  - Example: $C = \{0, 10, 110\}$

Comments (Cont.)

- Comment 1.25
  - If an r-ary code C is uniquely decodable, then it need not be instantaneous, but by Corollary 1.22 there must be an instantaneous r-ary code with the same word-lengths.
  - Examples: $C = \{0, 01, 11\}$ and $D = \{0, 10, 11\}$