

Coding and Information Theory

Overview

Chapter 1: Source Coding - C

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Quick Review of Last Lecture

- Uniquely Decodable Code
 - Compute C_n and C_∞
 - Compute counterexample to show a code C is not u.d.
 - Proof of the Sardinas-Patterson Theorem (Theorem 1.10)
- Instantaneous Code
 - Prefix Code
 - Constructing Instantaneous Code
 - Why is there a prefix code for a given set of word lengths or not?

1.5 Kraft's Inequality

- Theorem 1.20
 - There is an instantaneous r -ary code C with word-lengths l_1, \dots, l_q , if and only if

$$\sum_{i=1}^q \frac{1}{r^{l_i}} \leq 1. \quad (1.5)$$

1.6 McMillan's Inequality

- Theorem 1.21

- There is a uniquely decodable r -ary code C with word-lengths l_1, \dots, l_q , if and only if

$$\sum_{i=1}^q \frac{1}{r^{l_i}} \leq 1. \quad (1.6)$$

- Corollary 1.22

- There is an instantaneous r -ary code with word-lengths l_1, \dots, l_q , if and only if there is a uniquely decodable r -ary code with these word-lengths .

1.7 Comments on Kraft's and McMillan's Inequalities

- Comment 1.23
 - Theorems 1.20 and 1.21 do not say that an r -ary code with word-lengths l_1, \dots, l_q is instantaneous or uniquely decodable if and only if $\sum r^{-l_i} \leq 1$
 - Examples: $C = \{0, 01, 011\}$ and $C = \{0, 01, 001\}$
- Comment 1.24
 - Theorems 1.20 and 1.21 assert that if $\sum r^{-l_i} \leq 1$ then there exist codes with these parameters which are instantaneous and uniquely decodable.
 - Example: $C = \{0, 10, 110\}$

Comments (Cont.)

- Comment 1.25
 - If an r -ary code C is uniquely decodable, then it need not be instantaneous, but by Corollary 1.22 there must be an instantaneous r -ary code with the same word-lengths.
 - Examples: $C = \{0, 01, 11\}$ and $D = \{0, 10, 11\}$