

Homework Assignment #1
Fundamentals of Probability Exercises

Your Name: _____

A. (40 points) Multiple Choice Questions: Please select one best answer for each question

1. Two cards are drawn at random from a pack of 52 cards. The probability that both are spades is

- (a) $1/15$ (b) $2/17$ (c) $1/17$ (d) $2/15$

2. A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls. The probability that it is red is

- (a) $2/5$ (b) $4/15$ (c) $1/3$ (d) $2/3$

3. In the above problem, the probability that the ball is red or white is

- (a) $2/5$ (b) $2/3$ (c) $7/15$ (d) $1/5$

4. Three light bulbs are chosen at random from 15 bulbs out of which 5 are defective. The probability that exactly one is defective is

- (a) $24/91$ (b) $45/91$ (c) $67/91$ (d) none of these

5. In the above problem, the probability that none of the three bulbs is defective is

- (a) $13/91$ (b) $67/91$ (c) $55/97$ (d) $24/91$

6. Let A and B be two events with $P(A) = 3/8$, $P(B) = 3/8$, and $P(A \cap B) = 1/8$. The value of $P(A \cup B)$ is

- (a) $5/8$ (b) $1/3$ (c) $1/2$ (d) $3/8$

7. The probability of a 4 turning up at least once in two tosses of a fair die is

- (a) $11/36$ (b) $17/36$ (c) $13/36$ (d) $19/36$

8. A class has 12 boys and 4 girls. If three students are selected at random from the class, the probability that they are all boys is

- (a) $19/28$ (b) $3/28$ (c) $17/28$ (d) $11/28$

9. A box contains 7 red marbles and 3 white marbles. Three marbles are drawn from the box one after the other. The probability that the first two are red and the third is white is

- (a) $1/12$ (b) $2/9$ (c) $7/40$ (d) $13/40$

10. The expectation of the sum of points in tossing a pair of fair dice is

- (a) 9 (b) 5 (c) 6 (d) 7

11. In the above problem, the variance is

- (a) $37/6$ (b) $35/12$ (c) $35/6$ (d) none of these

12. A random variable X has the following density function:

$$f(x) = \begin{cases} \frac{1}{4} & -2 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Var(X) is

- (a) $1/3$ (b) $2/3$ (c) 1 (d) $4/3$

13. A random variable X has $E(X) = 2$, $E(X^2) = 8$. The standard deviation is

- (a) 1 (b) 2 (c) 4 (d) 10

B. (6 points) In a bolt factory, machines A, B, and C manufacture 25%, 35%, and 40% of the total of their output, respectively. Out of them, 5%, 4%, and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B, and C?

C. (6 points) A discrete random variable X assumes 10 values: 2, 3, 3, 4, 5, 5, 6, 6, 6, 7 with equal probability. Find the mean and variance of X .

D. (6 points) A discrete random variable X assumes 10 values: 2, 3, 3, 4, 5, 5, 6, 6, 6, 7 with the probability mass function.

x_i	2	3	3	4	5	5	6	6	6	7
$P(X = x_i)$	0.05	0.1	0.1	0.05	0.15	0.15	0.2	0.1	0.05	0.05

Find the mean and variance of X .