Homework 12: (Chapter 6 Error-correcting Codes)

Chapter 6 Exercises: 6.7, 6.9, 6.11, 6.14, 6.15 (26 points)

Exercise 6.7 (6%)
Let C be the extended binary Hamming code $\overline{H_7}$ (see Exercise 6.4). Find how many vectors $v \in V$ are covered by the spheres $S_t(\mathbf{u})$, where $\mathbf{u} \in C$, and hence show that this code is not perfect.

Exercise 6.9 (8%)
Find upper bounds for $A_3(n, 3)$ corresponding to those given for $A_2(n, 3)$ in Example 6.20. What does Hamming's sphere-packing bound imply about $A_2(n, 4)$ and $A_2(n, 5)$?

Exercise 6.11 (4%)
Find a lower bound for $A_3(n, 3)$.

Exercise 6.14 (3%)
Find all the code-words obtained in the above way from the Hadamard matrix H in Example 6.26. Do they form a linear code?

Exercise 6.15 (5%)
Construct a Hadamard matrix of order 8, and hence a Hadamard code of length 8. What is its rate? How many errors does it correct, and how many does it detect?