Homework 11: (Chapter 6 Error-correcting Codes)

Exercises: 6.1, 6.2, 6.3, 6.4 (16 points)

Exercise 6.1 (4%)
Prove that if C and C' are linear codes contained in V, then the codes $C \cap C'$ and $C + C' = \{u + u' \mid u \in C, u' \in C'\}$ are also linear. Under what circumstances is the code $C \cup C'$ linear?

Exercise 6.2 (4%)
Find the code-word in H_7 representing the information digits 1101, and show how an error in its 6th symbol is corrected. What happens if there are errors in the 4th and 6th symbols?

Exercise 6.3 (4%)
List all the codewords in the binary Hamming code H_7 (Example 6.5), and use Lemma 6.8 to verify that the minimum distance is 3.

Exercise 6.4 (4%)
Show that if C is a binary linear code of minimum distance d, then the extended code $\bar{C}$ has minimum distance d or $d + 1$ as d is even or odd. List the elements of the extended binary Hamming code $\overline{H_7}$, and find its minimum distance.