Homework 8: (Chapter 5 Using an Unreliable Channel)

Exercises: 5.1, 5.2, 5.4, 5.5, 5.8, 5.9, 5.10 (30 points)

Exercise 5.1 (2%)
How many different decision rules are there for a given information channel?

Exercise 5.2 (4%)
Calculate Pr_E, where the channel Γ and the input A are as in Example 4.5 (a BSC with P = 0.8 and p = 0.9), and Δ is the ideal observer rule.

Exercise 5.4 (4%)
If $u \in A^n$ where $|A| = r$, and $0 \leq i \leq n$, then how many words $v \in A^n$ have Hamming distance d(u, v) = i? Check that these numbers, for i = 0, 1, ..., n, add up to $|A^n|$.

Exercise 5.5 (4%)
How large can a subset $C \subseteq Z_2^3$ be, if d(u, v) ≥ 2 for all $u \neq v$ in C? Describe geometrically all the subsets attaining this bound. What is the analogous bound for subsets of Z_2^n?

Exercise 5.8 (6%)
Let Γ be the BEC, with P > 0, and let the input probabilities be p, $\bar{p}$ with 0 < p < 1. Show how to use the binary repetition code R_n to send information through Γ so that $Pr_E \longrightarrow 0$ as $n \longrightarrow \infty$.

Exercise 5.9 (6%)
A binary channel Γ always transmits 0 correctly but transmits 1 as 1 or 0 with probabilities P and $Q = \bar{P}$, where 0 < P < 1. Write down the channel matrix and describe the maximum likelihood rule. If the input probabilities of 0 and 1 are p and $\bar{p}$, find Pr_E. To improve reliability, 0 and 1 are encoded as 000 and 111. Describe the resulting maximum likelihood rule; is it the same as (i) majority decoding, (ii) nearest neighbor decoding? Find the resulting rate and error-probability. What happens if instead we use the binary repetition code R_n, and let $n \longrightarrow \infty$?

Exercise 5.10 (Modified) (4%)
The binary repetition code R_n, of odd length n = 2t + 1, is used to encode messages transmitted through a BSC Γ in which each digit has probabilities P and Q (= $\bar{P}$) of correct or incorrect transmission, and $P > 1/2$. Show that in this case the maximum likelihood rule, majority decoding and nearest neighbor decoding all give the same decision rule Δ.