

Homework 7: (Chapter 4 Information Channel)**Exercise 4.2 (6%)**

Let Γ be the BSC. Find necessary and sufficient conditions (on p and P) for Γ to satisfy

(i) $Q_{00} < Q_{10}$ and $Q_{01} < Q_{11}$;

(ii) $Q_{00} > Q_{10}$ and $Q_{01} < Q_{11}$;

(iii) $Q_{00} < Q_{10}$ and $Q_{01} > Q_{11}$.

What do these conditions mean, from the point of view of the receiver?

Exercise 4.3 (4%)

Prove equation (4.7), that $H(A, B) = H(B) + H(A | B)$. What interpretation of the equivocation $H(A | B)$ does this imply?

Exercise 4.6 (3%)

Give an example of an information channel Γ and an input A for which $H(B) < H(A)$, where B is the resulting output.

Exercise 4.7 (10%)

Calculate the system entropies where Γ is the binary erasure channel (BEC), introduced in §4.1, and the input probabilities of 0 and 1 are p and \bar{p} . Show that this channel satisfies (4.9) and (4.10).

$$H(\mathcal{B} | \mathcal{A}) \leq H(\mathcal{B}), \quad (4.9)$$

$$H(\mathcal{A} | \mathcal{B}) \leq H(\mathcal{A}), \quad (4.10)$$