Homework #2 (Chapter 1: Source Coding)

Chapter 1 Exercises: 1.2, 1.3, 1.5, 1.8, 1.9, 1.11 and two additional questions

1. (10%) Consider the following table where a source S with 4 symbols has been encoded in binary codes with 0 and 1.

Symbol s_i	Code 1	Code 2	Code 3	Code 4
s_1	00	0	0	0
s_2	01	1	10	01
s_3	10	00	110	011
s_4	11	11	111	0111

Please identify which code is uniquely decodable and which code is instantaneous (or prefix)? (Note: you do not need to prove)

2. (6%) Consider the following table where two binary codes are given for a source S with four symbols s_i and corresponding probabilities p_i.

Symbol s_i	Probability p_i	Code 1	Code 2
s_1	0.5	00	0
s_2	0.25	01	10
s_3	0.125	00	110
s_4	0.125	11	111

(1) Please compute the average word length for Code 1.
(2) Please compute the average word length for Code 2.

Exercise 1.2 (10%): Construct the sets C_n and C_∞ for the ternary code C = {02, 12, 120, 20, 21}. Do the same for C = {02, 12, 120, 21}.

Exercise 1.3 (6%): Determine whether or not the codes C = {02, 12, 120, 20, 21} and C = {02, 12, 120, 21} considered in Exercise 1.2 are uniquely decodable. If C is not uniquely decodable, find a code-sequence which can be decoded in at least two ways.

Exercise 1.5 (4%): A code C exhibits non-unique decodability in the form 012120.120 = 01.212.01.20; find an element of $C \cap C_\infty$.

Exercise 1.8 (6%): Show that the binary code C = {0, 01, 011, 111} is uniquely decodable; how should the receiver react on receiving a sequence starting 0111...1...?

Exercise 1.9 (4%)
Is this also true for the code D = {0, 10, 110, 111}, the reverse of the code C in Exercise 1.8?

Exercise 1.11 (6%): Find an instantaneous ternary code with word-lengths 1, 2, 3, 3, 4. Is there one with word-lengths 1, 1, 2, 2, 2, 2?