CS 4410

Automata, Computability, and Formal Language

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Chapter 8

Properties of Context-Free Languages

1. Two Pumping Lemmas
   - A Pumping Lemma for Context-Free Languages
   - A Pumping Lemma for Linear Language
2. Closure Properties and Decision Algorithms for Context-Free Languages
   - Closure of Context-Free Languages
   - Some Decidable Properties of Context-Free Languages

Closure of Context-Free Languages

Theorem 8.5: Let L_1 be a context-free language and L_2 be a regular language. Then $L_1 \cap L_2$ is context-free.

Example 8.7: The language $L=\{a^n b^n : n \geq 0, n \neq 100\}$ is context-free.

Closure of Context-Free Languages

Example 8.8: Show that the language

$$L=\{w \in \{a,b,c\}^* : n_a(w)=n_b(w)=n_c(w)\}$$

is not context-free.

Elementary Questions about Context-Free Languages

- Given a context-free language L and an arbitrary string w, is there an algorithm to determine whether or not w is in L?
- Given a context-free language L, is there an algorithm to determine if L is empty?
- Given a context-free language L, is there an algorithm to determine if L is infinite?
- Given two context-free grammars G_1 and G_2, is there an algorithm to determine if $L(G_1) = L(G_2)$?

A Membership Algorithm for Context-Free Languages

- The combination of Theorems 5.2 and 6.5 confirms the existence of a membership algorithm for context-free languages
  - By Theorem 5.2, exhaustive parsing is guaranteed to give the correct result for any context-free grammar that contains neither λ-productions nor unit-productions
  - By Theorem 6.5, such a grammar can always be produced if the language does not include λ
- Alternatively, a npda to accept the language can be constructed as established by Theorem 7.1

Determining Whether a Context-Free Language is Empty

- Theorem 8.6 confirms the existence of an algorithm to determine if a context-free language L(G) is empty
  - For simplicity, assume that λ is not in L(G)
  - Apply the algorithm for removing useless symbols and productions
  - If the start symbol is found to be useless, then L(G) is empty; otherwise, L(G) contains at least one string

Determining Whether a Context-Free Language is Infinite

- Theorem 8.7 confirms the existence of an algorithm to determine if a context-free language L(G) is infinite
  - Apply the algorithms for removing λ-productions, unit-productions, and useless productions
  - If G has a variable A for which there is a derivation that allows A to produce a sentential form xAy, then L(G) is infinite. Otherwise, L(G) is finite

Determining Whether Two Context-Free Languages are Equal

- Given two context-free grammars G_1 and G_2, is there an algorithm to determine if $L(G_1) = L(G_2)$?
- If the languages are finite, the answer can be found by performing a string-by-string comparison
- However, for general context-free languages, <u>no algorithm exists to determine equality</u>