CS 4410

Automata, Computability, and Formal Language

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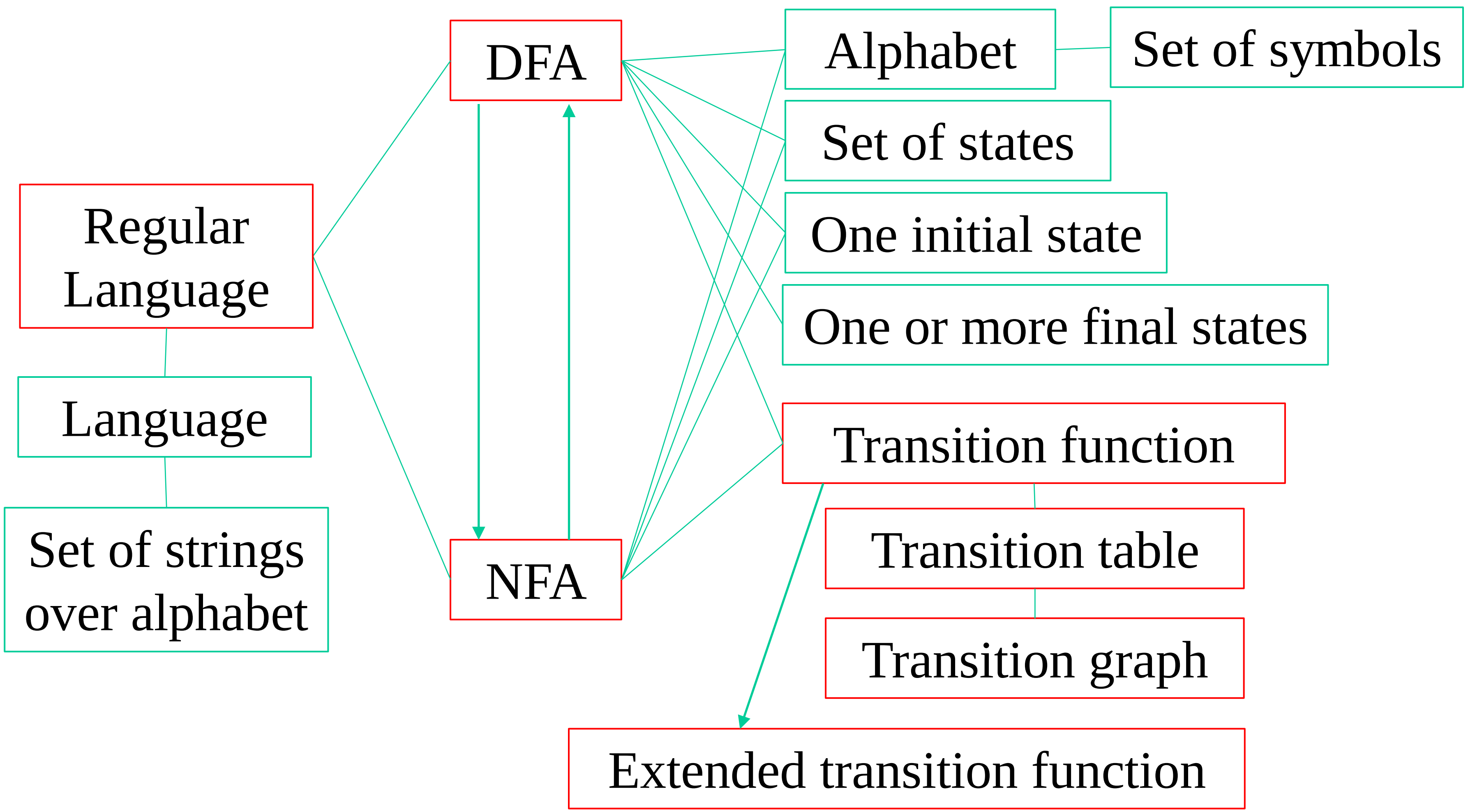
Review of Chapter 2
Concept Map



Chapter 3

Regular Languages and Regular Grammars

1. Regular Expressions
   - Formal Definition of a regular Expression
   - Languages Associated with Regular Expressions
2. Connection Between Regular Expressions and Regular Languages
   - Regular Expressions Denote Regular Languages
   - Regular Expressions for Regular Languages
   - Regular Expressions for Describing Simple Patterns
3. Regular Grammars
   - Right- and Left-Linear Grammars
   - Right-Linear Grammars Generate Regular Languages
   - Right-Linear Grammars for Regular Languages
   - Equivalence Between Regular Languages and Regular grammars

Learning Objectives

At the conclusion of the chapter, the student will be able to:

- Identify the language associated with a regular expression
- Find a regular expression to describe a given language
- Construct a nondeterministic finite automaton to accept the language denoted by a regular expression
- Use generalized transition graphs to construct a regular expression that denotes the language accepted by a given finite automaton
- Identify whether a particular grammar is regular
- Construct regular grammars for simple languages
- Construct a nfa that accepts the language generated by a regular grammar
- Construct a regular grammar that generates the language accepted by a finite automaton

Regular Expression

Definition 3.1

Let Σ be a given alphabet. Then

1. $\varnothing$, λ, and $a \in \Sigma$ are all regular expressions. These are called primitive regular expressions.
2. If r_1, r_2 and r are regular expressions, so are $r_1 + r_2$, $r_1 \bullet r_2$, r^*, and (r).
3. A string is a regular expression if and only if it can be derived from the primitive regular expressions by a finite number of applications of the rules in (2).

Example 3.1

For $\Sigma = \{a, b, c\}$,
the string $(a+b \bullet c)^* \bullet (c + \varnothing)$ is a regular expression,
but, the string $(a+b+)$ is not.

Languages Associated with Regular Expressions

Definition 3.2

The language L(r) denoted by any regular expression r is defined by the following rules.

1. $\varnothing$ is a regular expression denoting the empty set,
2. λ is a regular expression denoting $\{\lambda\}$,
3. For every $a \in \Sigma$, a is a regular expression denoting $\{a\}$.

If r_1, r_2 and r are regular expressions, then

4. $L(r_1 + r_2) = L(r_1) \cup L(r_2)$
5. $L(r_1 \bullet r_2) = L(r_1)\, L(r_2)$
6. $L((r)) = L(r)$
7. $L(r^*) = (L(r))^*$

Precedence rule

Star-closure: *

Concatenation: •

Union: +

Note: • can be omitted.

Sample Regular Expressions and Associated Languages

Regular Expression	Language
(ab)*	$\{ (ab)^n, n \geq 0 \}$
a + b	{ a, b }
(a + b)*	{ a, b }* (in other words, any string formed with a and b)
a(bb)*	{ a, abb, abbbb, abbbbbb, … }
a*(a + b)	{ a, aa, aaa, …, b, ab, aab, … } (Example 3.2)
(aa)*(bb)*b	{ b, aab, aaaab, …, bbb, aabbb, … } (Example 3.4)
(0 + 1)*00(0 + 1)*	Binary strings containing at least one pair of consecutive zeros

Languages and Regular Expressions

Example 3.2 $L(a^* \bullet (a+b)) = ?$

Languages and Regular Expressions

Example 3.3

Let r = (a+b)* (a + bb). L(r) = ?

Languages and Regular Expressions

Example 3.4

Let $r = (aa)^* (bb)^* b$. $L(r) = ?$

Languages and Regular Expressions

Example 3.5 For $\Sigma = \{0, 1\}$, give a regular expression r such that

$L(r) = \{ w \in \{0,1\}^*$: w has at least one pair of consecutive zeros$\}$

Languages and Regular Expressions

Example 3.6 Find a regular expression for the language

$L(r) = \{ w \in \{0,1\}^*: w \text{ has no pair of consecutive zeros}\}$

We say the two regular expressions are equivalent if they denote the same language.

Regular Expressions Denote Regular Languages

Theorem 3.1: Let *r* be a regular expression. Then there exists some nondeterministic finite accepter that accepts L(*r*). Consequently, L(*r*) is a regular language.

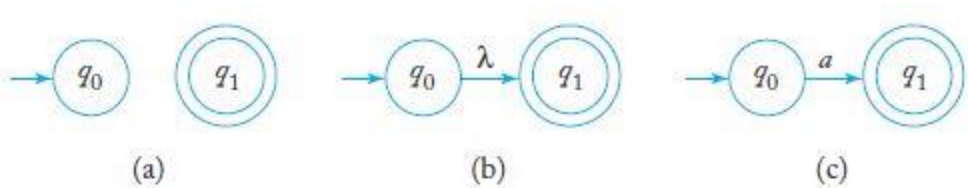


FIGURE 3.1 (a) nfa accepts $\varnothing$. (b) nfa accepts $\{\lambda\}$. (c) nfa accepts $\{a\}$.

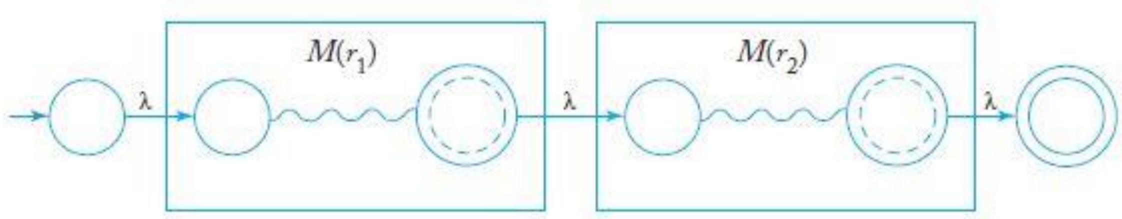


FIGURE 3.4 Automaton for $L(r_1 r_2)$.