Math 2300, Spring 2020

Proofs using the Principle of **Strong** Mathematical Induction

Principle of Strong Mathematical Induction (Epp, p. 268)

Let P(n) be a property that is defined for integers n, and let a and b be fixed integers with $a \leq b$. Suppose the following two statements are true:

1. P(a), P(a + 1), . . . , and P(b) are all true. (basis step)
2. For any integer $k \geq b$, if P(i) is true for all integers i from a through k, then P(k + 1) is true. (inductive step)

Then the statement
for all integers $n \geq a$, P(n) is true.

(The supposition that P(i) is true for all integers i from a through k is called the inductive hypothesis. Another way to state the inductive hypothesis is to say that P(a), P(a + 1), . . . , P(k) are all true.)

Dr. Martin's 5 step method to prove P(n) for all integers $n \geq a$.

1. **Base Cases:** $\boldsymbol{P(a), P(a+1), \dots, P(b)}$

   If it is an equation be sure to check both sides separately.

2. Write down the Induction Hypothesis: n=i

   Where k is a particular, but arbitrary integer greater than or equal to b.

   Let $\boldsymbol{k \in \mathbb{Z} \ni k \geq b}$ **and suppose** $\boldsymbol{P(i)\ \forall i \in \mathbb{Z} \ni a \leq i \leq k}$

3. Write down what needs to be proved: n = k + 1

   We must show that P(k+1)

4. Show P(a) through P(k) implies P(k+1) by clear and convincing argument, usually starts:

   Consider P(k+1)

5. Conclusion

   By PSMI P(n) $\boldsymbol{\forall n \in \mathbb{Z} \ni n \geq a}$