

Math 2300
Section 3.1

A **predicate** is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables. The **domain of a predicate variable** is the set of all values that may be substituted in place of the variable.

If $P(x)$ is a predicate and x has domain D , the **truth set of $P(x)$** is the set of all elements of D that make $P(x)$ true when they are substituted for x . The truth set of $P(x)$ is denoted $\{x \in D \mid P(x)\}$.

Let $Q(x)$ be a predicate and D the domain of x . A **universal statement** is a statement of the form " $\forall x \in D, Q(x)$." It is defined to be true if, and only if, $Q(x)$ is true for every x in D . It is defined to be false if, and only if, $Q(x)$ is false for at least one x in D . A value for x for which $Q(x)$ is false is called a counterexample to the universal statement.

Let $Q(x)$ be a predicate and D the domain of x . An **existential statement** is a statement of the form " $\exists x \in D$ such that $Q(x)$." It is defined to be true if, and only if, $Q(x)$ is true for at least one x in D . It is false if, and only if, $Q(x)$ is false for all x in D .