Math 2300

Section 4.3, 4.4

Theorem 4.3.5 Unique Factorization of Integers Theorem (Fundamental Theorem of Arithmetic)

Given any integer $n > 1$, there exist a positive integer k, distinct prime numbers $p_1, p_2, . . . , p_k$, and positive integers $e_1, e_2, . . . , e_k$ such that

$$n = p_1^{e1} p_2^{e2} p_3^{e3} . . . p_k^{ek}$$

and any other expression for n as a product of prime numbers is identical to this except, perhaps, for the order in which the factors are written.

Theorem 4.4.1 The Quotient-Remainder Theorem

Given any integer n and positive integer d, there exist unique integers q and r such that $\boldsymbol{n = dq + r}$ **and** $\boldsymbol{0 \le r < d}$**.**

Definition of **div** and **mod**

Given an integer n and a positive integer d,

> **n div d** = the integer **quotient** obtained when n is divided by d, and
>
> **n mod d** = the nonnegative integer **remainder** obtained when n is divided by d.

Symbolically, if n and d are integers and $d > 0$, then

$$n \text{ div } d = q \quad \text{and} \quad n \text{ mod } d = r \quad \Leftrightarrow \quad n = dq + r$$

where q and r are integers and $0 \le r < d$.