Second Order Linear Recurrence Relations with Constant Coefficients

Five steps:

0. Check the recurrence is SOLRRCC

A second-order linear recurrence relation with constant coefficients is a recurrence relation of the form:

$$a_k = Aa_{k-1} + Ba_{k-2} + f(k)$$

for all integers $k \geq$ some fixed integer, where A and B are fixed real numbers.

1. Find the Particular Solution p_n

We consider three cases:

- Constant: example $f(k) = -1$; guess $p_n = c$ and solve for c
- Linear: example $f(k) = k$; guess $p_n = a+bn$ and solve for a and b
- Exponential: example $f(k) = 6^k$; guess $p_n = a6^n$ and solve for a

2. Find the homogeneous solution q_n

Let q_n be the solution to the homogenous part of the recurrence

$$a_k = Aa_{k-1} + Ba_{k-2}$$

for all integers $k \geq$ some fixed integer, where A and B are fixed real numbers.

3. Write $a_n = p_n + q_n$

4. Solve for C and D, using the initial conditions.

Two linear equations in two variables using linear algebra.

5. Write down the final formula, specifying where it holds (e.g. $\forall n \in \mathbb{Z} \ni n \geq 0$).