

## Math 2300, Spring 2017

### Proofs using the Principle of **Strong** Mathematical Induction

#### **Principle of Strong Mathematical Induction** (Epp, p. 268)

Let  $P(n)$  be a property that is defined for integers  $n$ , and let  $a$  and  $b$  be fixed integers with  $a \leq b$ . Suppose the following two statements are true:

1.  $P(a)$ ,  $P(a + 1)$ ,  $\dots$ , and  $P(b)$  are all true. (basis step)
2. For any integer  $k \geq a$ , if  $P(i)$  is true for all integers  $i$  from  $a$  through  $k$ , then  $P(k + 1)$  is true. (inductive step)

Then the statement  
for all integers  $n \geq a$ ,  $P(n)$  is true.

(The supposition that  $P(i)$  is true for all integers  $i$  from  $a$  through  $k$  is called the inductive hypothesis. Another way to state the inductive hypothesis is to say that  $P(a)$ ,  $P(a + 1)$ ,  $\dots$ ,  $P(k)$  are all true.)

Dr. Martin's 5 step method to prove  $P(n)$  for all integers  $n \geq a$ .

#### 1. **Base Cases: $P(a)$ , $P(a + 1)$ , $\dots$ , $P(b)$**

If it is an equation be sure to check both sides separately.

#### 2. Write down the Induction Hypothesis: $n=i$

Where  $k$  is a particular, but arbitrary integer greater than or equal to  $b$ .

**Let  $k \in \mathbb{Z} \ni k \geq b$  and suppose  $P(i) \forall i \in \mathbb{Z} \ni a \leq i \leq k$**

#### 3. Write down what needs to be proved: $n = k + 1$

**We must show that  $P(k+1)$**

#### 4. Show $P(a)$ through $P(k)$ implies $P(k+1)$ by clear and convincing argument, usually starts:

**Consider  $P(k+1)$**

#### 5. Conclusion

**By PSMI  $P(n) \forall n \in \mathbb{Z} \ni n \geq a$**