# Second Order Linear Homogeneous Recurrence Relations with Constant Coefficients (Epp section 5.8)

Five steps:

**0. Check the recurrence is SOLHRRCC**

A second-order linear homogeneous recurrence relation with constant coefficients is a recurrence relation of the form:

$$a_k = Aa_{k-1} + Ba_{k-2}$$

for all integers k ≥ some fixed integer, where A and B are fixed real numbers with $B \neq 0$.

**1. Find the Characteristic Polynomial**

Let A and B be real numbers. A recurrence relation of the form

$a_k = Aa_{k-1} + Ba_{k-2}$ for all integers $k \geq 2$

is satisfied by the sequence

$1, t, t^2, t^3, \ldots, t^n, \ldots,$

where t is a nonzero real number, if, and only if, t satisfies the equation

$$t^2 - At - B = 0 \text{ (this is the characteristic polynomial)}$$

**2. Find the roots of the Characteristic Polynomial**

Factor or use quadratic formula

**3. Use Theorems to find the formula**

Distinct Roots

If the characteristic equation

$t^2 - At - B = 0$

has two distinct roots r and s, then $a_0, a_1, a_2, \ldots$ is given by the explicit formula

$$a_n = Cr^n + Ds^n,$$

where C and D are the numbers whose values are determined by the values $a_0$ and $a_1$.

Repeated Roots

If the characteristic equation

$t^2 - At - B = 0$

has single (real) root r, then $a_0, a_1, a_2, \ldots$ is given by the explicit formula

$$a_n = Cr^n + Dnr^n,$$

where C and D are the numbers whose values are determined by the values $a_0$ and $a_1$.

**4. Solve for C and D, using the initial conditions.**

Two linear equations in two variables using linear algebra.

**5. Write down the final formula, specifying where it holds (e.g. $\forall n \in \mathbb{Z} \ni n \geq 0$).**