Math 2300, Spring 2016 – Discrete Structures
Sample Problems
Material Covered Since Quiz 2

For 1-2 use the theorem on the handout.

1. $a_k = 4a_{k-1} - 9, \quad k \geq 1, \quad a_0 = 1$

2. $a_k = 2a_{k-1} + 3a_{k-2} + 5^k, \quad k \geq 2, \quad a_0 = -2, a_1 = 1$

3. Let $A = \{1,2,3\}$ and $B = \{x, y\}$.
   a. List the elements of $A \times B$.
   b. List the elements of the power set of A: $\mathscr{P}(A)$.

4. Let *A, B*, and *C* be sets. Prove that $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$.

5. Let *A, B*, and *C* be sets. Use set identities to prove that $(A - B) - C = A - (B \cup C)$.

6. Assume that all sets are subsets of a universal set *U* and prove that for all sets *A* and *B*, if $A \subseteq B$, then $A \cap B^c = \emptyset$.

7. Let $A = \{1,2,3,4\}$, draw a directed graph of a relation on *A* that is:
   a. Reflexive, but not symmetric and not transitive.
   b. Symmetric and transitive, but not reflexive.

8. Let $A = \{2,3,5\}$ and $B = \{2,6,15\}$ and let *R* be the "divides" relation from *A* to *B*:
$$\forall (x,y) \in A \times B, xRy \Leftrightarrow x \mid y$$
   a. Explicitly state which ordered pairs are in *R*.
   b. Explicitly state which ordered pairs are in R^{-1}.

9. Let R be the relation defined on Z as follows:
$$\forall m,n \in Z, mRn \Leftrightarrow 5 \mid (m-n)$$
   Prove that R is an equivalence relation.