Math 2300, Spring 2011 – Discrete Structures
Quiz 3 – Sample Problems

1. Solve the recurrence relation:
$$a_k = 3a_{k-1} + 6^k,\ k \ge 1, \text{ given } a_0 = 1.$$

2. Let $A = \{1,2,3\}$ and $B = \{x, y\}$.
   a. List the elements of $A \times B$.
   b. List the elements of the power set of A: $\mathscr{P}(A)$.

3. Let *A, B*, and *C* be sets. Prove that $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$.

4. Let *A, B*, and *C* be sets. Use set identities to prove that $(A - B) - C = A - (B \cup C)$.

5. Assume that all sets are subsets of a universal set *U* and prove that for all sets *A* and *B*, if $A \subseteq B$, then $A \cap B^c = \varnothing$.

6. Let $A = \{1,2,3,4\}$, draw a directed graph of a relation on *A* that is:
   a. Reflexive, but not symmetric and not transitive.
   b. Symmetric and transitive, but no reflexive.

7. Let $A = \{2,3,5\}$ and $B = \{2,6,15\}$ and let *R* be the "divides" relation from *A* to *B*:
$$\forall (x, y) \in A \times B, xRy \Leftrightarrow x \mid y$$
   a. Explicitly state which ordered pairs are in *R*.
   b. Explicitly state which ordered pairs are in R^{-1}.

8. Let R be the relation defined on Z as follows:
$$\forall m, n \in Z, mRn \Leftrightarrow 5 \mid (m - n)$$
   a. Prove that R is an equivalence relation.
   b. List (no proof required) the distinct equivalence classes of R.

9. Consider the subset relation, ⊆, on the set $\mathscr{P}(\{a, b, c\})$. That is for all *U* and *V* in $\mathscr{P}(\{a, b, c\})$:
$$U \subseteq V \Leftrightarrow \forall x, x \in U \Rightarrow x \in V$$
   Draw the Hasse diagram for this relation.

10. How many integers from 1 through 100 must you pick to be sure of getting one that is divisible by 5?

11. a. Draw the complete graph on seven vertices, K_7.
    b. Draw the bipartite graph $K_{3,4}$.

12. Does this graph (not shown – but this gives you an idea of a type of question) have and Euler Circuit? Why or why not? If not can you remove one edge to result in a subgraph with an Euler Circuit? (Specify which edge.) Label the vertices and describe the path that forms the circuit.