Some previous projects

- Twenty Questions
- Library Search Assistant
- Euclid's Game
- FedEx on the Go
- Battleship Game
- Line-Following Robot
- Connect Four
- CS Course Chooser

Some previous projects

- Learning Checkers
- Shoot 'em Up
- Agent Using Genetic Algorithm
- Guess Who
- Color Memory Game
- TicTac Chat
- Eight Queens
- Super Mario Bros. AI

Some previous projects

- Blackjack with various AI solution
- Intelligent Pong
- Wine without Whining
- Neural Net OCR
- The Sherpa – hike recommender
- Virtual Pet
- Sudoku
- Lego Mindstorms color sorter

Some previous projects

- Maze Solving
- Spam Filtering
- Intelligent Crew Scheduler
- Machine Translation: English/Japanese
- Cross-Country Game
- Chatbot
- Turing Test

Search

Dr. Melanie Martin
CS 4480

Chapter 3

- Search
  - Problem-solving agents
  - Problem types
  - Problem formulation
  - Example problems
  - Basic search algorithms

Problem-solving agents

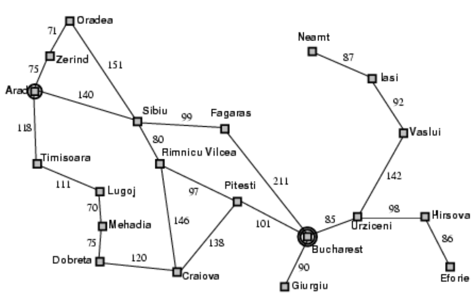
```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, percept)
   if seq is empty then do
       goal ← FORMULATE-GOAL(state)
       problem ← FORMULATE-PROBLEM(state, goal)
       seq ← SEARCH( problem)
   action ← FIRST(seq)
   seq ← REST(seq)
   return action
```

Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania

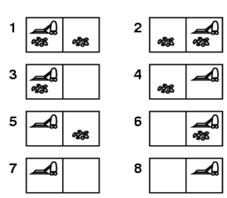


Problem types

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - often interleave} search, execution
- Unknown state space → exploration problem

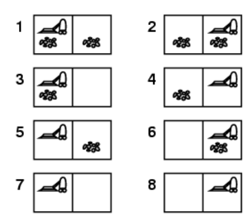
Example: vacuum world

- Single-state, start in #5.
  Solution?



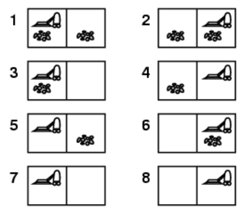
Example: vacuum world

- Single-state, start in #5.
  Solution? *[Right, Suck]*
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*}
  Solution?



Example: vacuum world

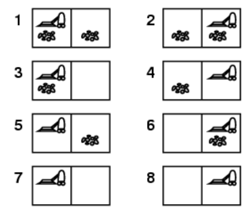
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*}
  Solution?
  [Right,Suck,Left,Suck]



- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean]*, i.e., start in #5 or #7
    Solution?

Example: vacuum world

- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*}
  Solution?
  [Right,Suck,Left,Suck]



- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean]*, i.e., start in #5 or #7
    Solution? *[Right, **if** dirt **then** Suck]*

Single-state problem formulation

A problem is defined by four items:

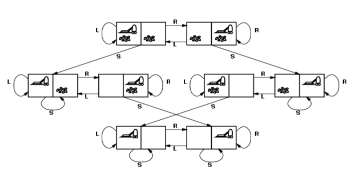
1. initial state e.g., "at Arad"
2. actions or successor function *S(x)* = set of action–state pairs
   - e.g., *S(Arad) = {<Arad → Zerind, Zerind>, … }*
3. goal test, can be
   - explicit, e.g., *x* = "at Bucharest"
   - implicit, e.g., *Checkmate(x)*
4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - *c(x,a,y)* is the step cost, assumed to be ≥ 0

- A solution is a sequence of actions leading from the initial state to a goal state

Selecting a state space

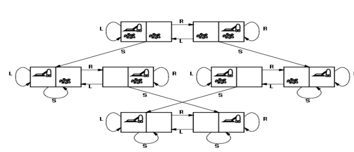
- Real world is absurdly complex
  → state space must be abstracted for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

Vacuum world state space graph



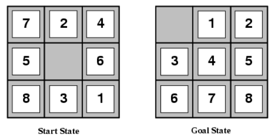
- states?
- actions?
- goal test?
- path cost?

Vacuum world state space graph



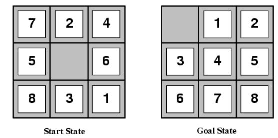
- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

Example: The 8-puzzle



- states?
- actions?
- goal test?
- path cost?

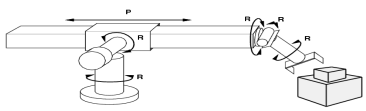
Example: The 8-puzzle



- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

[Note: optimal solution of *n*-Puzzle family is NP-hard]

Example: robotic assembly



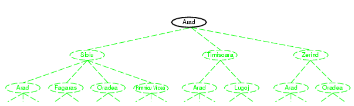
- states?: real-valued coordinates of robot joint angles parts of the object to be assembled
- actions?: continuous motions of robot joints
- goal test?: complete assembly
- path cost?: time to execute

Tree search algorithms

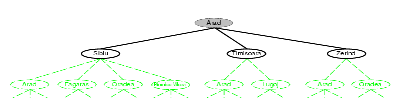
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
     if there are no candidates for expansion then return failure
     choose a leaf node for expansion according to strategy
     if the node contains a goal state then return the corresponding solution
     else expand the node and add the resulting nodes to the search tree
```

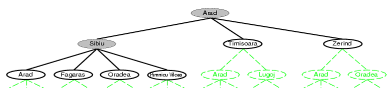
Tree search example



Tree search example



Tree search example

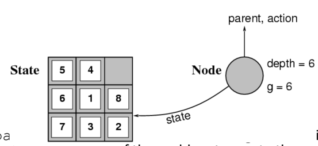


Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
       fringe ← INSERTALL(EXPAND(node, problem), fringe)
---------------------------------------------------------------------
function EXPAND( node, problem) returns a set of nodes
   successors ← the empty set
   for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
       s ← a new NODE
       PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
       PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
       DEPTH[s] ← DEPTH[node] + 1
       add s to successors
   return successors
```

Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



- The Expa ious fields and using the SuccessorFn of the problem to create the corresponding states.

Search strategies

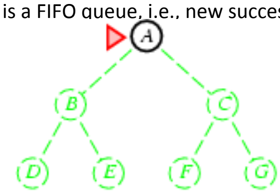
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b*: maximum branching factor of the search tree
  - *d*: depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



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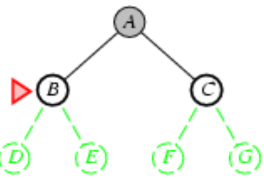

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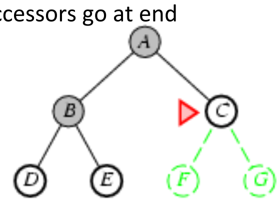


Properties of breadth-first search

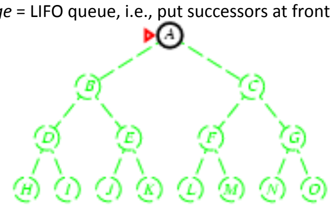
- Complete? Yes (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots+b^d + b(b^d-1) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)
- Space is the bigger problem (more than time)

Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *frontier* = priority queue ordered by path cost g(n)
- Equivalent to breadth-first if step costs all equal
- Complete? Yes, if step cost ≥ ε
- Time? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where C^* is the cost of the optimal solution
- Space? # of nodes with g ≤ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of *g(n)*

Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



Depth-first search

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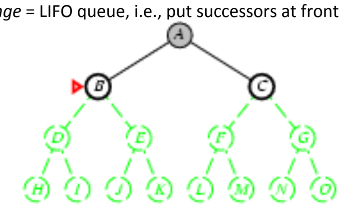

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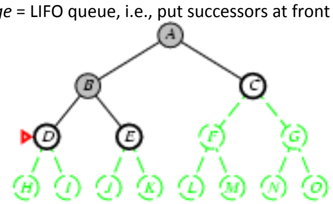

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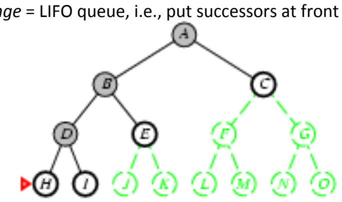

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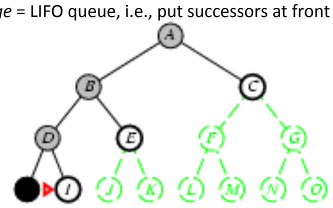

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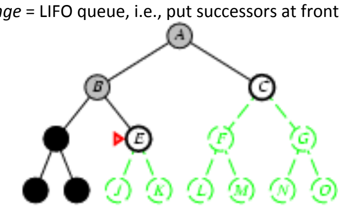

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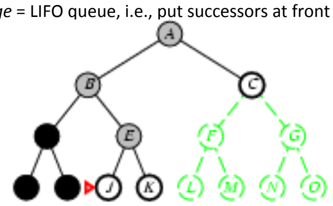

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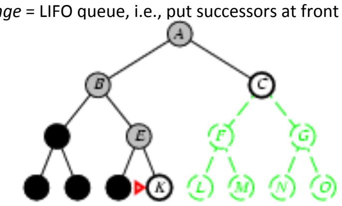

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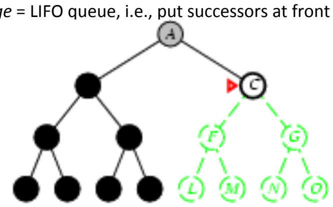

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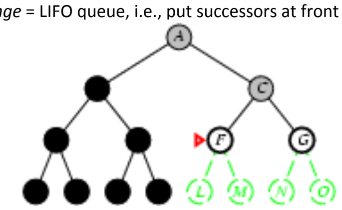

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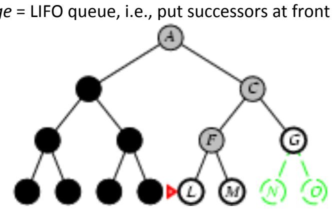

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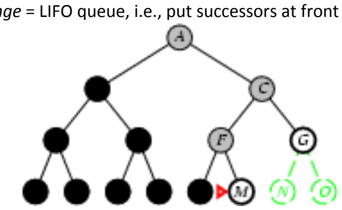


Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path

    → complete in finite spaces
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space!
- Optimal? No

Depth-limited search

= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors

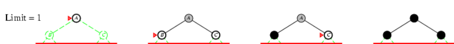
```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

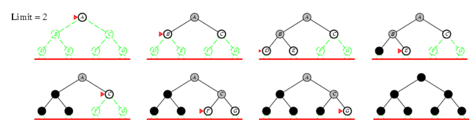
function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution, or failure
  inputs: *problem*, a problem
  for *depth* ← 0 **to** ∞ **do**
    result ← DEPTH-LIMITED-SEARCH(*problem*, *depth*)
    if *result* ≠ cutoff **then return** *result*

Iterative deepening search *l* =0

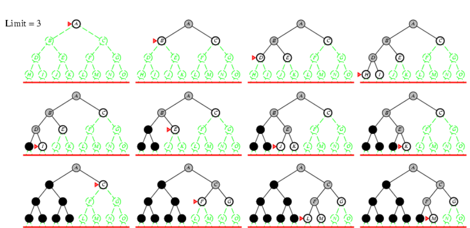
Iterative deepening search *l* =1



Iterative deepening search *l* =2



Iterative deepening search *l* =3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:
  $N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$
- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:
  $N_{IDS} = (d+1)b^0 + d\, b^{\wedge 1} + (d-1)b^{\wedge 2} + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$
- For *b = 10, d = 5,*
  - N_{DLS} = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111
  - N_{IDS} = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456
- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

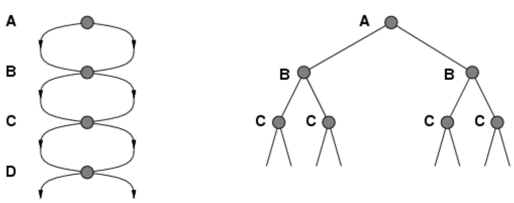
- Complete? Yes
- Time? $(d+1)b^0 + d\,b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!



Graph search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
       if STATE[node] is not in closed then
           add STATE[node] to closed
           fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

Outline

- Chapter 3 – Informed Search
  - Best-first search
  - Greedy best-first search
  - A^* search
  - Heuristics
- Chapter 4 – Coming Soon
  - Local search algorithms
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search
  - Genetic algorithms

Best-first search

- Idea: use an evaluation function $f(n)$ for each node
  - estimate of "desirability"

  → Expand most desirable unexpanded node

- Implementation:
  Order the nodes in frontier in decreasing order of desirability

- Special cases:
  - greedy best-first search
  - A^* search

Heuristic

- Problem solving by experimental methods
  - Trial and error
- Heuristic function h(n)
  - Takes node as input
  - Depends only on state of node
  - Estimated cost of cheapest path from node n to a goal node
  - Numerical estimate of the "goodness" of a state

Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic)
  = estimate of cost from *n* to *goal*

- e.g., $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest

- Greedy best-first search expands the node that appears to be closest to goal

Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →

- Time? $O(b^m)$, but a good heuristic can give dramatic improvement

- Space? $O(b^m)$ -- keeps all nodes in memory

- Optimal? No

A^* search

- Idea: avoid expanding paths that are already expensive

- Evaluation function $f(n) = g(n) + h(n)$

- $g(n)$ = cost so far to reach *n*
- $h(n)$ = estimated cost from *n* to goal
- $f(n)$ = estimated total cost of path through *n* to goal

Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node *n*, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from *n*.

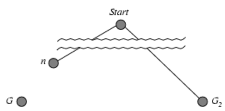
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic

- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

- Theorem: If $h(n)$ is admissible, A^* using TREE-SEARCH is optimal

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ $f(G_2) = g(G_2) > g(G) = f(G)$

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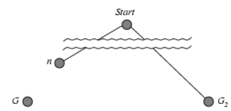
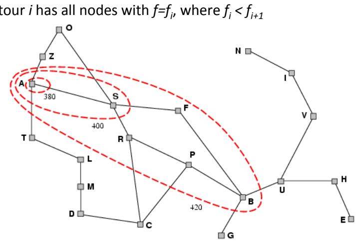


- $f(G_2)$ $> f(G)$ from previous slide
- $h(n)$ $\le h^*(n)$ since h is admissible – h*(n) is true cost
- $g(n) + h(n)$ $\le g(n) + h^*(n)$
- $f(n)$ $\le f(G)$ $\le f(G_2)$

Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion

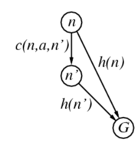
Optimality of A*

- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*, the estimated cost of reaching the goal from *n* is no greater than the step cost of getting to *n'* plus the estimated cost of reaching the goal from *n'*:
  $h(n) \le c(n,a,n') + h(n')$
- If *h* is consistent, we have
  $f(n') = g(n') + h(n')$
  $= g(n) + c(n,a,n') + h(n')$
  $\ge g(n) + h(n)$
  $= f(n)$



- i.e., *f(n)* is non-decreasing along any path.
- Theorem: If *h(n)* is consistent, A* using `GRAPH-SEARCH` is optimal

Properties of A*

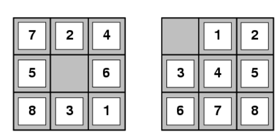
- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



- $h_1(S)$ = ?
- $h_2(S)$ = ?

Admissible heuristics

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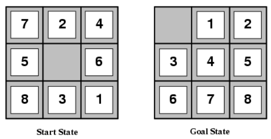

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution