

Logic

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Based on slides from
<http://aima.eecs.berkeley.edu/2nd-ed/slides-ppt/>

Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg (P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \vee \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law ($\wedge$ over $\vee$) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Resolution algorithm

- Proof by contradiction, i.e., show $KB \wedge \neg \alpha$ unsatisfiable

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function PL-RESOLUTION( $KB, \alpha$ ) returns true or false
  clauses  $\leftarrow$  the set of clauses in the CNF representation of  $KB \wedge \neg \alpha$ 
  new  $\leftarrow \{\}$ 
  loop do
    for each  $C_i, C_j$  in clauses do
      resolvents  $\leftarrow$  PL-RESOLVE( $C_i, C_j$ )
      if resolvents contains the empty clause then return true
    new  $\leftarrow$  new  $\cup$  resolvents
  if new  $\subseteq$  clauses then return false
  clauses  $\leftarrow$  clauses  $\cup$  new
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Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \wedge \neg P_{1,2}$

