Uncertainty

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Based on slides from

http://aima.eecs.berkeley.edu/2nd-ed/slides-ppt/

Outline

- Uncertainty
- Probability
- Syntax and Semantics
- Inference
- Independence and Bayes' Rule

Uncertainty

Let action A_t = leave for airport $_t$ minutes before flight
Will A_t get me there on time?

Problems:

1. partial observability (road state, other drivers' plans, etc.)
2. noisy sensors (traffic reports)
3. uncertainty in action outcomes (flat tire, etc.)
4. immense complexity of modeling and predicting traffic

Hence a purely logical approach either

1. risks falsehood: "A_{25} will get me there on time", or
2. leads to conclusions that are too weak for decision making:

"A_{25} will get me there on time if there's no accident on the bridge and it doesn't rain and my tires remain intact etc etc."

(A_{1440} might reasonably be said to get me there on time but I'd have to stay overnight in the airport …)

Methods for handling uncertainty

- Default or nonmonotonic logic:
  - Assume my car does not have a flat tire
  - Assume A_{25} works unless contradicted by evidence
- Issues: What assumptions are reasonable? How to handle contradiction?

- Rules with fudge factors:
  - $A_{25} \mapsto_{0.3}$ get there on time
  - *Sprinkler* $\mapsto_{0.99}$ *WetGrass*
  - *WetGrass* $\mapsto_{0.7}$ *Rain*
- Issues: Problems with combination, e.g., *Sprinkler* causes *Rain*??

- Probability
  - Model agent's degree of belief
  - Given the available evidence,
  - A_{25} will get me there on time with probability 0.04

Probability

Probabilistic assertions summarize effects of

- laziness: failure to enumerate exceptions, qualifications, etc.
- ignorance: lack of relevant facts, initial conditions, etc.

Subjective probability:

- Probabilities relate propositions to agent's own state of knowledge

  e.g., $P(A_{25} \mid \text{no reported accidents}) = 0.06$

These are not assertions about the world

Probabilities of propositions change with new evidence:

e.g., $P(A_{25} \mid \text{no reported accidents, 5 a.m.}) = 0.15$

Making decisions under uncertainty

Suppose I believe the following:

$P(A_{25}$ gets me there on time | …) = 0.04

$P(A_{90}$ gets me there on time | …) = 0.70

$P(A_{120}$ gets me there on time | …) = 0.95

$P(A_{1440}$ gets me there on time | …) = 0.9999

- Which action to choose?

  Depends on my preferences for missing flight vs. time spent waiting, etc.
  - Utility theory is used to represent and infer preferences
  - Decision theory = probability theory + utility theory

Syntax

- Basic element: random variable
  - Refers to a part of the world whose status is initially unknown
  - Assigns a numerical value to each outcome of an experiment
- Similar to propositional logic: possible worlds defined by assignment of values to random variables.
- Boolean random variables
  e.g., *Cavity* (do I have a cavity?)
- Discrete random variables
  e.g., *Weather* is one of *<sunny,rainy,cloudy,snow>*
- Domain values must be exhaustive and mutually exclusive
- Elementary proposition constructed by assignment of a value to a random variable: e.g., *Weather = sunny*, *Cavity = false* (abbreviated as ¬*cavity*)
- Complex propositions formed from elementary propositions and standard logical connectives e.g., *Weather = sunny* ∨ *Cavity = false*
- Also continuous: time, distance, weight

Syntax

- Atomic event: A complete specification of the state of the world about which the agent is uncertain

  E.g., if the world consists of only two Boolean variables *Cavity* and *Toothache*, then there are 4 distinct atomic events:

  $Cavity = false \wedge Toothache = false$

  $Cavity = false \wedge Toothache = true$

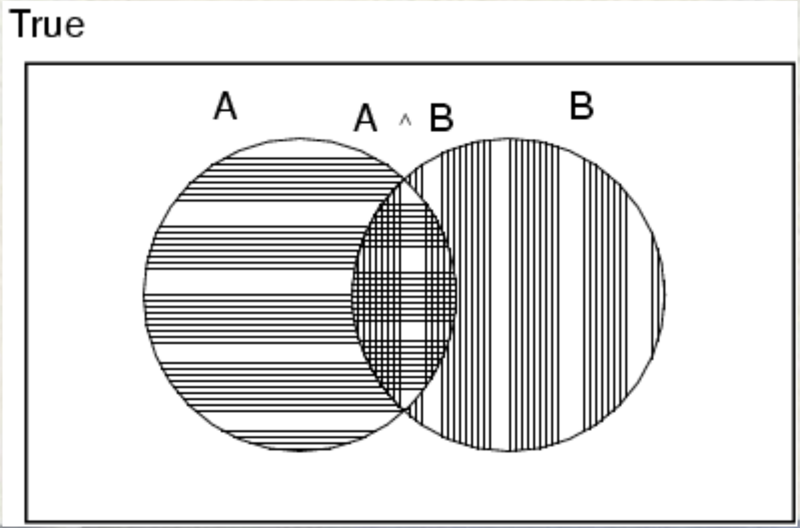
  $Cavity = true \wedge Toothache = false$

  $Cavity = true \wedge Toothache = true$

- Atomic events are mutually exclusive and exhaustive
- AKA: Sample space is the set of elementary outcomes

Axioms of probability

- For any propositions *A*, *B* (Events)
  - $0 \leq P(A) \leq 1$
  - $P(true) = 1$ and $P(false) = 0$
  - $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$



Prior probability

- Prior or unconditional probabilities of propositions
  e.g., P(*Cavity* = true) = 0.1 and P(*Weather* = sunny) = 0.72 correspond to belief prior to arrival of any (new) evidence
- Probability distribution gives values for all possible assignments:
  P(*Weather*) = <0.72,0.1,0.08,0.1> (normalized, i.e., sums to 1)
- Joint probability distribution for a set of random variables gives the probability of every atomic event on those random variables
  P(*Weather,Cavity*) = a 4 × 2 matrix of values:

Weather =	sunny	rainy	cloudy	snow
Cavity = true	0.144	0.02	0.016	0.02
Cavity = false	0.576	0.08	0.064	0.08

- Every question about a domain can be answered by the joint distribution
- Note these are intersections

Conditional probability

- Conditional or posterior probabilities
  e.g., P(*cavity* | *toothache*) = 0.8
  i.e., given that *toothache* is all I know

- (Notation for conditional distributions:
  P(*Cavity* | *Toothache*) = 2-element vector of 2-element vectors)

- If we know more, e.g., *cavity* is also given, then we have
  P(*cavity* | *toothache,cavity*) = 1

- New evidence may be irrelevant, allowing simplification, e.g.,
  P(*cavity* | *toothache, sunny*) = P(*cavity* | *toothache*) = 0.8
- This kind of inference, sanctioned by domain knowledge, is crucial