Adversarial Search

Dr. Melanie Martin

CS 4480

October 1, 2010

Based on slides from

http://aima.eecs.berkeley.edu/2nd-ed/slides-ppt/

Outline

- Optimal decisions
- α-β pruning
- Imperfect, real-time decisions

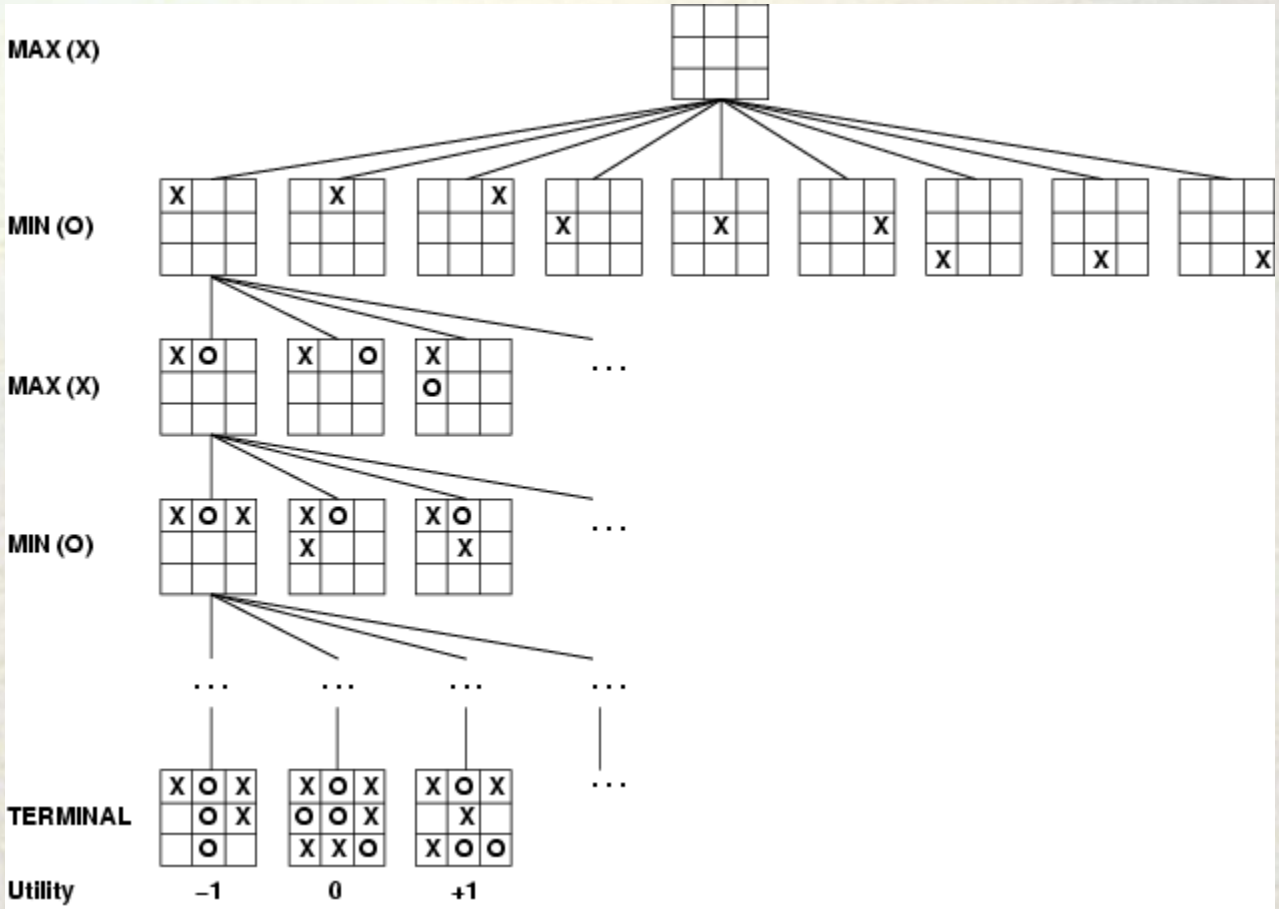
Games vs. search problems

- "Unpredictable" opponent → specifying a move for every possible opponent reply
- Time limits → unlikely to find goal, must approximate

Game as Search Problem

- Initial State
  - Board position and player to move
- Successor Function
  - Returns list of *(state, move)* pairs
    - Legal moves and resulting states
- Terminal (Goal) Test
  - When game is over
- Utility (Objective) Function
  - Assigns numeric outcome to terminal states
    - E.g. +1, -1, 0 for win, lose, draw

Game tree (2-player, deterministic, turns)

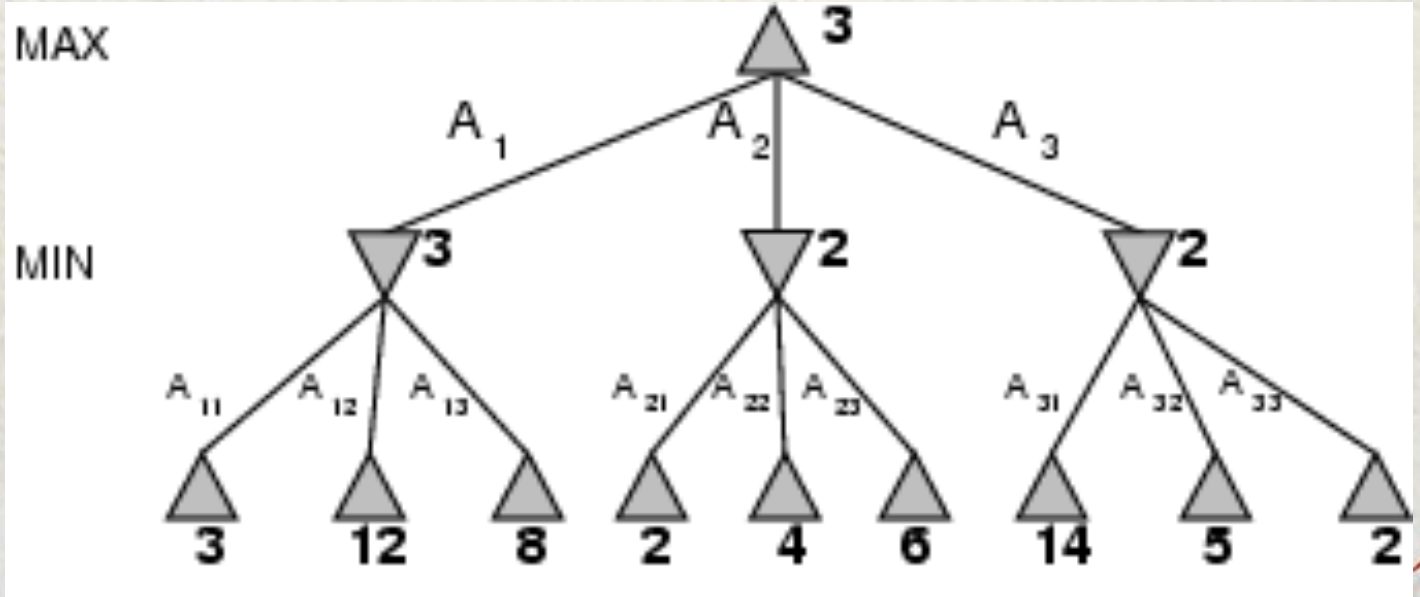


Optimal Strategy

- Leads to outcomes as good as any other strategy when playing an infallible opponent
- Tree where max takes a turn and min takes a turn is ONE MOVE DEEP made up two half-moves - each half move is called a ply

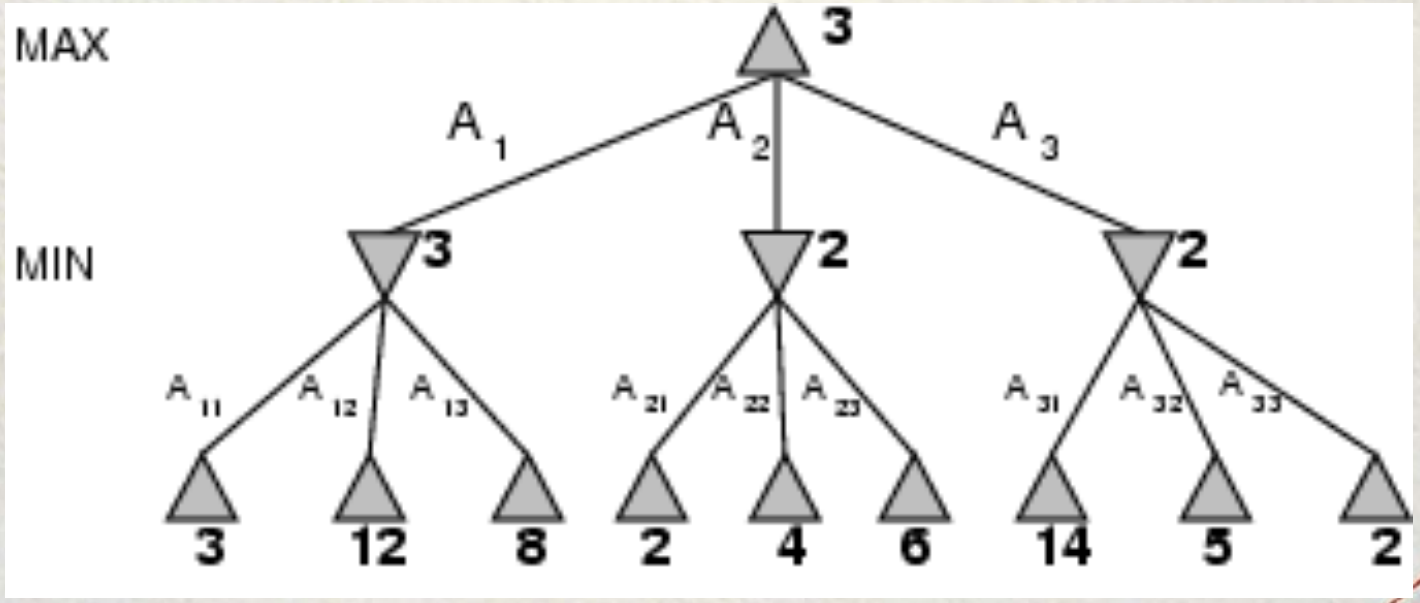
Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value = best achievable payoff against best play
- E.g., 2-ply game:



Minimax

- Minimax(node) is utility for max of being in corresponding state
- Max prefers a state with maximum value
- Min prefers a state with minimum value



Minimax algorithm

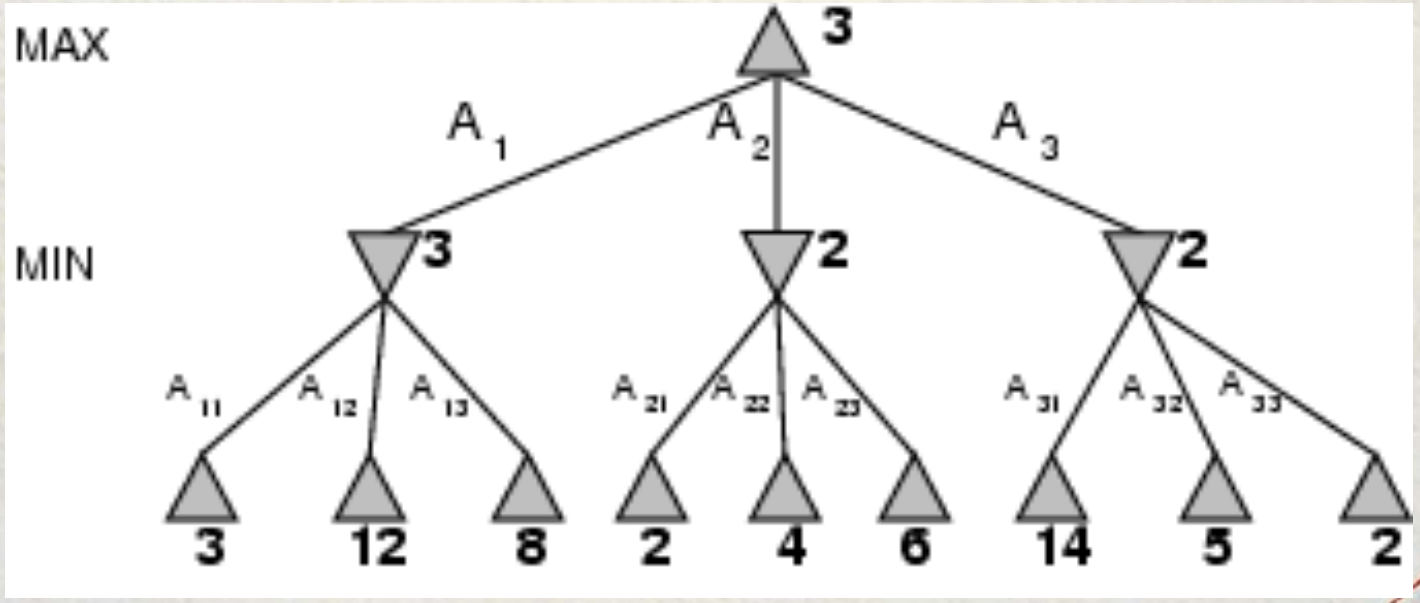
```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

Minimax

- Recursion proceeds to leaves and based on utility function assigns minimax values at the level above and so on



Properties of minimax

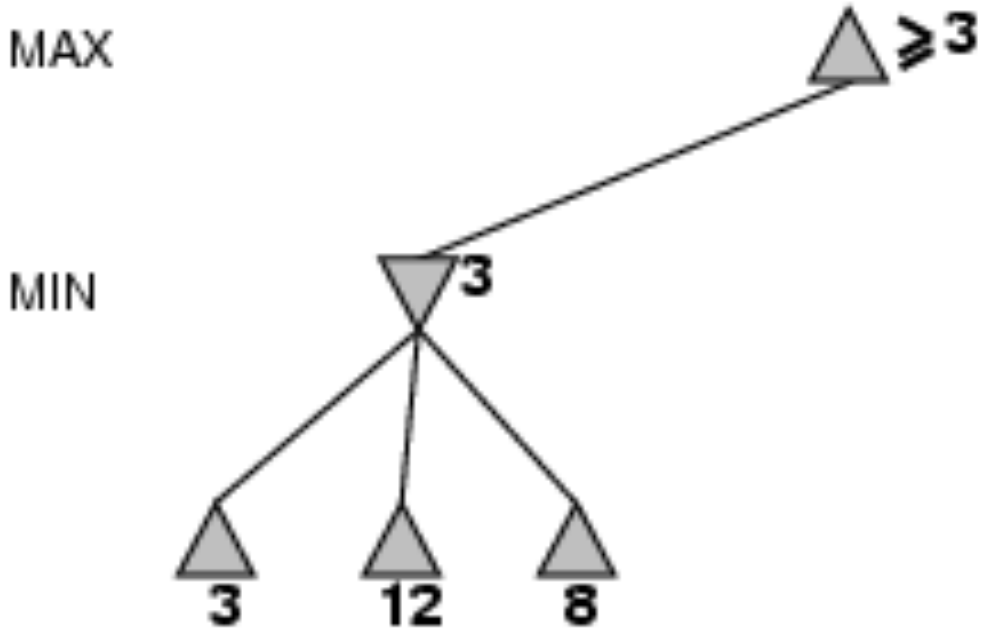
- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)

- For chess, b ≈ 35, m ≈100 for "reasonable" games
  → exact solution completely infeasible

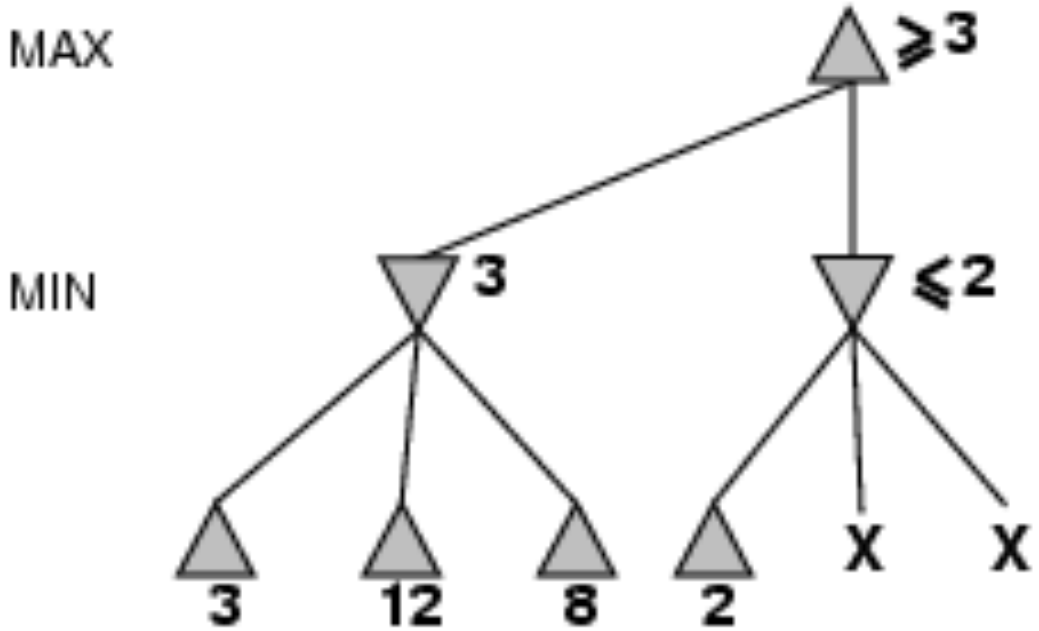
Problem

- Number of game states exponential in number of moves
- Can cut in half with pruning
  - Still exponential
  - Get rid of braches that can't influence final decision

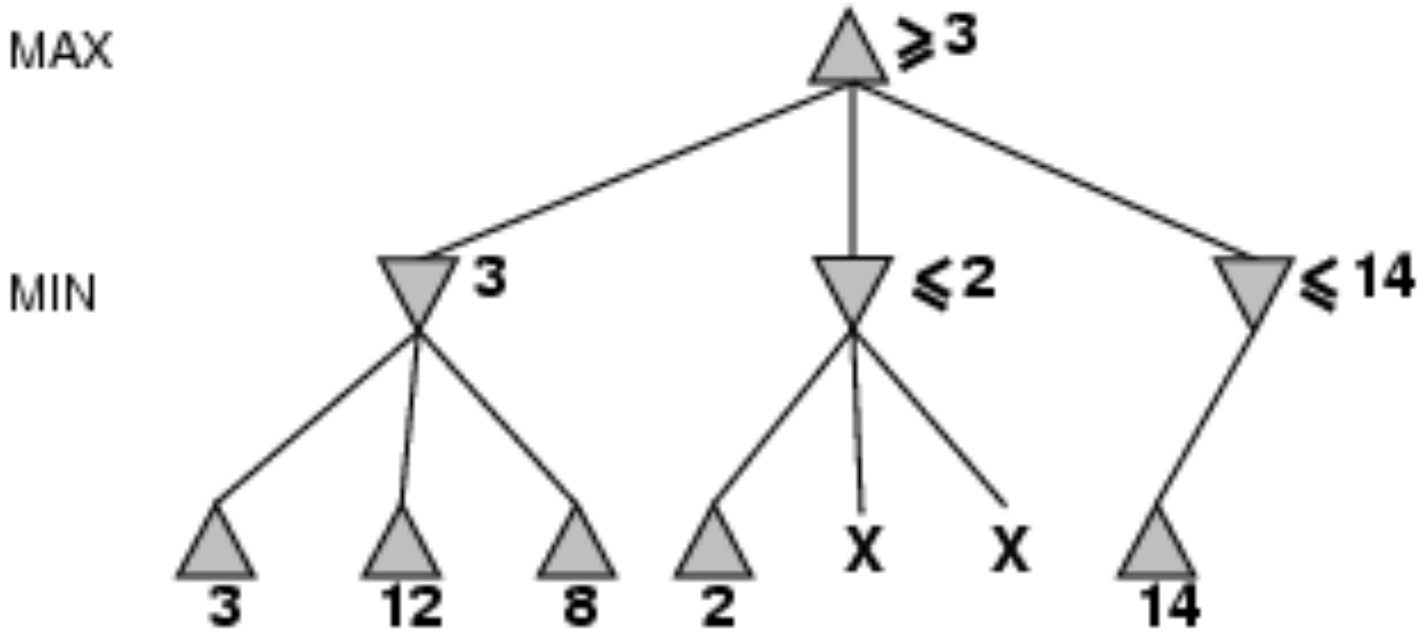
α-β pruning example



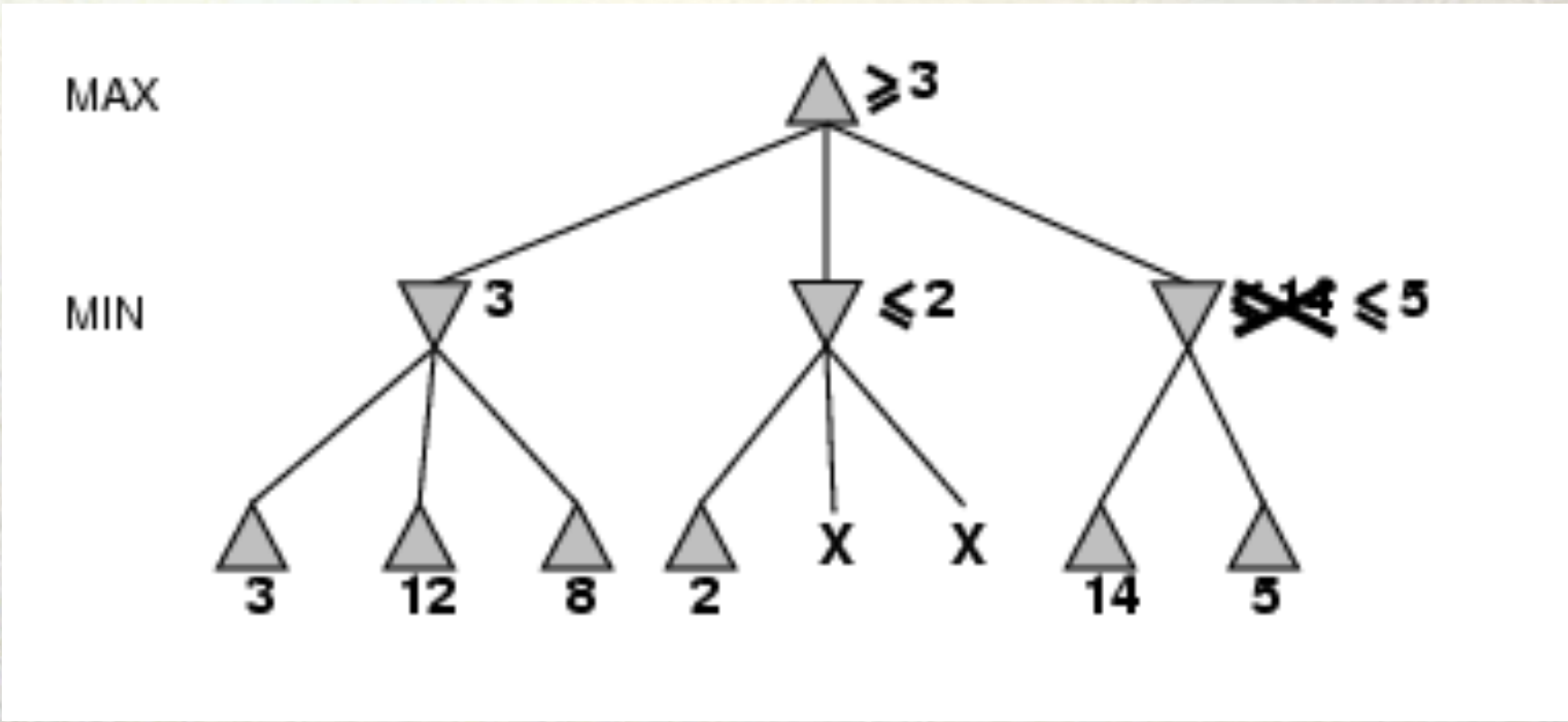
α-β pruning example



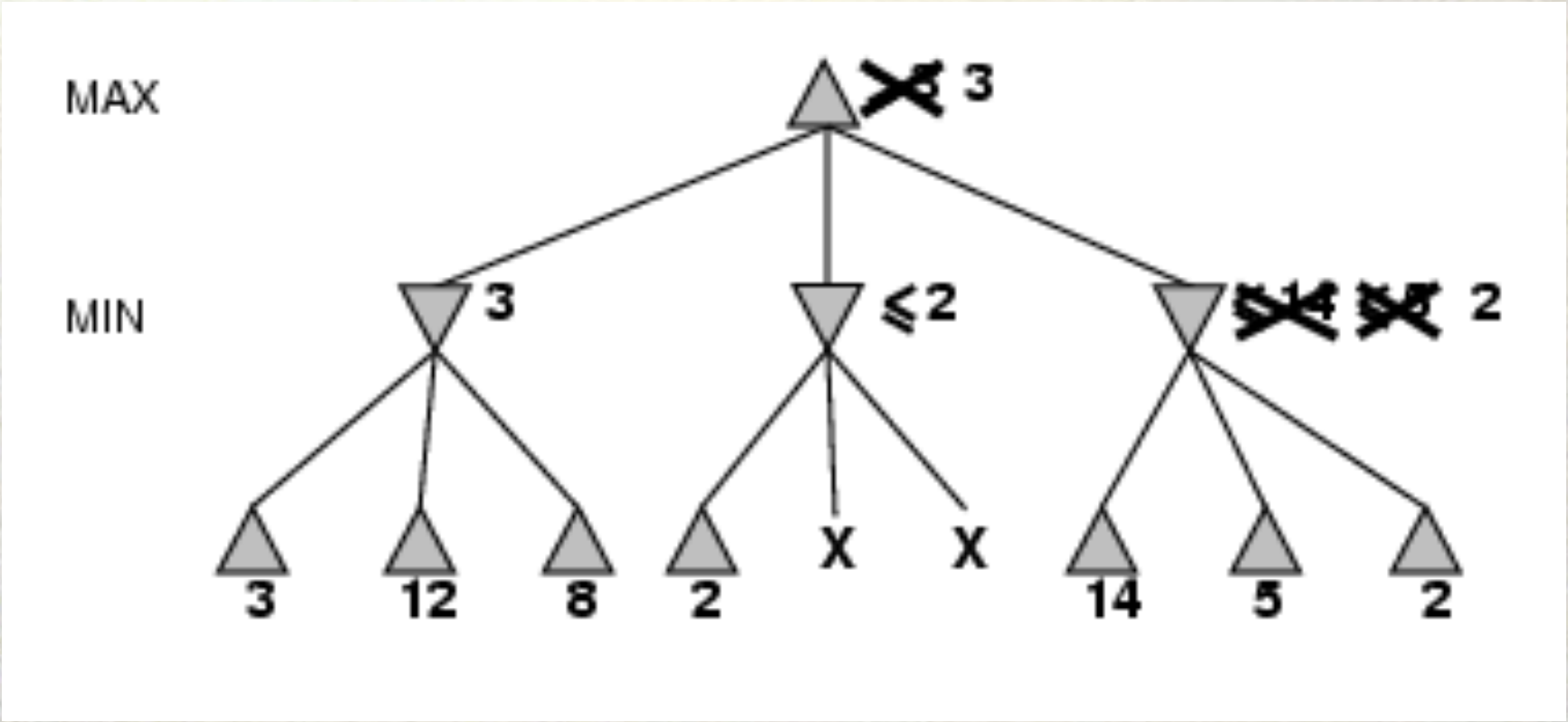
α-β pruning example



α-β pruning example



α-β pruning example

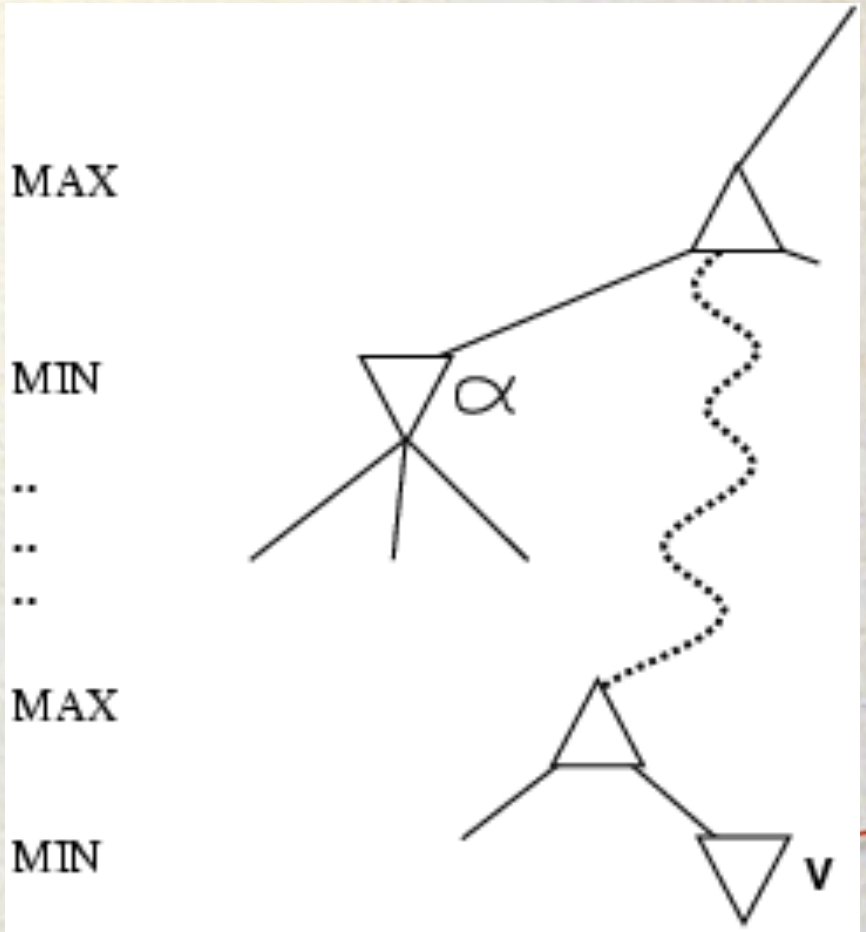


Properties of α-β

- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*
- If *v* is worse than α, *max* will avoid it
  → prune that branch
- Define β similarly for *min*



The α-β algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game

  $v \leftarrow$ MAX-VALUE($state, -\infty, +\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

The α-β algorithm

```
function MIN-VALUE(state, α, β) returns a utility value
    inputs: state, current state in game
            α, the value of the best alternative for MAX along the path to state
            β, the value of the best alternative for MIN along the path to state

    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← +∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s, α, β))
        if v ≤ α then return v
        β ← MIN(β, v)
    return v
```

Resource limits

Suppose we have 100 secs, explore 10^4 nodes/sec

→ 10^6 nodes per move

Standard approach:

- cutoff test:

  e.g., depth limit

- evaluation function

  = estimated desirability of position

  Usually a heuristic function

Evaluation functions

- For chess, typically linear weighted sum of features

  $$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g., $w_1 = 9$ with

  $f_1(s)$ = (number of white queens) – (number of black queens), etc.

Cutting off search

MinimaxCutoff is identical to *MinimaxValue* except

1. *Terminal?* is replaced by *Cutoff?*
2. *Utility* is replaced by *Eval*

Does it work in practice?

$b^m = 10^6$, b=35 → m=4

4-ply lookahead is a hopeless chess player!

- 4-ply ≈ human novice
- 8-ply ≈ typical PC, human master
- 12-ply ≈ Deep Blue, Kasparov

Deterministic games in practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.

»

- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

- Othello: human champions refuse to compete against computers, who are too good.

- Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

Summary

- Games are fun to work on!
- They illustrate several important points about AI
- perfection is unattainable → must approximate
- good idea to think about what to think about