First-Order Logic

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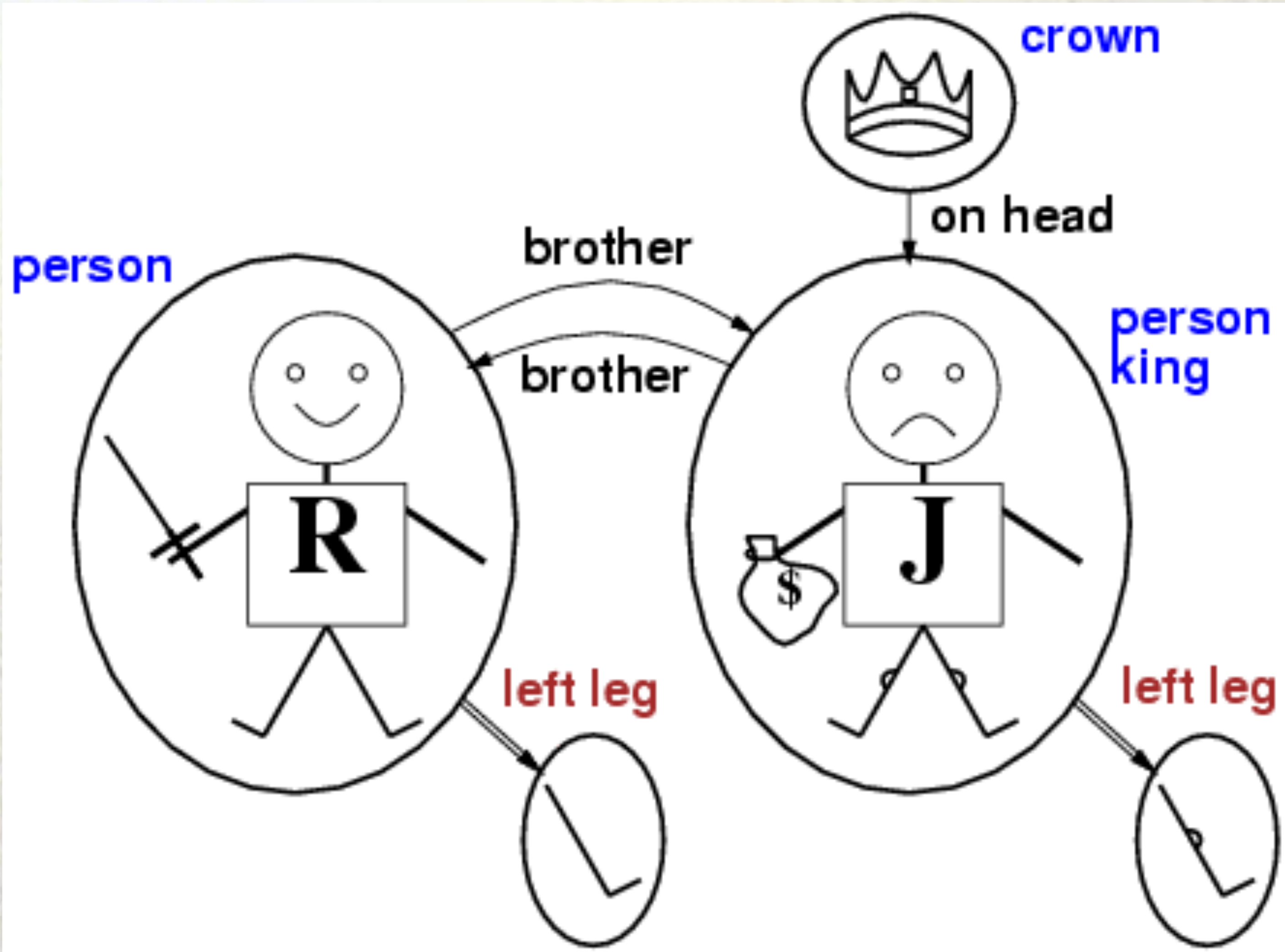
Based on slides from

http://aima.eecs.berkeley.edu/2nd-ed/slides-ppt/

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- E.g., definition of *Sibling* in terms of *Parent*:

  $\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \land \exists m,f\ \neg (m = f) \land Parent(m,x) \land Parent(f,x) \land Parent(m,y) \land Parent(f,y)]$


crown
on head
brother
brother
person
person
king
R
J
$
left leg
left leg

- Predicate of brotherhood:
  - {<R,J>,<J,R>}
- Predicate of being on: {<C,J>}
- Predicate of being a person:
  - {J,R}
- Predicate of being the king: {J}
- Predicate of being a crown: {C}
- Function for left legs: <{J,JLL},{R,RLL}>

Interpretation

- Specifies which objects, functions, and predicates are referred to by which constant symbols, function symbols, and predicate symbols.
- Under the intended interpretation:
- “richardI” refers to R; “johnII” refers to J; “crown” refers to the crown.
- “onHead”,”brother”,”person”,”king”, “crown”, “leftLeg”, “strong”

Lots of other possible interpretations

- 5 objects, so just for constants “richard” and “john” there are 25 possibilities
- Note that the legs don’t have their own names!
- “johnII” and “johnLackland” may be assigned the same object, J
- Also possible: “crown” and “johnII” refer to C (just not the intended interpretation)

Why isn't the "intended interpretation" enough?

- Vague notion.   What is intended may be ambiguous (and often is, for non-toy domains)
- Logically possible: square(x) ^ round(x). Your KB has to include knowledge that rules this out.

Determining truth values of FOPC sentences

- Assign meanings to terms:
  - “johnII” ← J; “leftLeg(johnII)”← JLL
- Assign truth values to atomic sentences
  - “brother(johnII,richardI)”
  - “brother(johnlackland,richardI)”
  - Both True, because <J,R> is in the set assigned “brother”
  - “strong(leftleg(johnlackland))”
  - True, because JLL is in the set assigned “strong”

Examples given the Sample Interpretation

- $\forall$ X,Y brother(X,Y) FALSE
- $\forall$ X,Y (person(X) ^ person(Y)) → brother(X,Y) FALSE
- $\forall$ X,Y (person(X) ^ person(Y) ^ ~(X=Y)) → brother(X,Y) TRUE
- $\exists$ X crown(X) TRUE
- $\exists$ X $\exists$ Y sister(X,Y) FALSE

Representational Schemes

- What are the objects, predicates, and functions? Keep in mind that you need to encode knowledge of specific problem instances and general knowledge.
- In practice, consider interpretations just to understand what the choices are. The world and interpretation are defined, or at least constrained, through the logical sentences we write.

Example Choice: Predicates versus Constants

- Rep-Scheme 1: Let's consider the world: D = {a,b,c,d,e}. green: {a,b,c}. blue: {d,e}. Some sentences that are satisfied by the intended interpretation:

green(a). green(b). blue(d).

~(∀ x green(x)). ∀ x green(x) v blue(x).

But what if we want to say that blue is pretty?

Choice:
Predicates versus Constants

- Rep-Scheme 2: The world: D = {a,b,c,d,e,green,blue}
  colorof:
  {<a,green>,<b,green>,<c,green>,<d,blue>,<e,blue>}
  pretty: {blue} notprimary: {green}

- Some sentences that are satisfied by the intended interpretation:

  colorOf(a,green). colorOf(b,green). colorOf(d,blue).

  ~(∀ X colorOf(X,green)).

  ∀ X colorOf(X,green) v colorOf(X,blue).

  pretty(blue). notprimary(green).

We have reified predicates blue and green: made them into objects

Using FOL

The kinship domain:

- Brothers are siblings

  $\forall x,y\ Brother(x,y) \Leftrightarrow Sibling(x,y)$

- One's mother is one's female parent

  $\forall m,c\ Mother(c) = m \Leftrightarrow (Female(m) \wedge Parent(m,c))$

- "Sibling" is symmetric

  $\forall x,y\ Sibling(x,y) \Leftrightarrow Sibling(y,x)$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

  `Tell`(KB,Percept([Smell,Breeze,None],5))
  `Ask`(KB,∃a BestAction(a,5))

- I.e., does the KB entail some best action at *t=5*?
- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)
- Given a sentence *S* and a substitution σ,
- *S*σ denotes the result of plugging σ into *S*; e.g.,
  S = Smarter(x,y)
  σ = {x/Hillary,y/Bill}
  *S*σ = Smarter(Hillary,Bill)
- `Ask`(KB,S) returns some/all σ such that KB ⊨ Sσ

Knowledge base for the wumpus world

- Perception
  - $\forall$t,s,b Percept([s,b,Glitter],t) $\Rightarrow$ Glitter(t)
- Reflex
  - $\forall$t Glitter(t) $\Rightarrow$ BestAction(Grab,t)

Deducing hidden properties

- $\forall x,y,a,b$ *Adjacent*([x,y],[a,b]) $\Leftrightarrow$ [a,b] $\in$ {[x+1,y], [x-1,y],[x,y+1],[x,y-1]}

Properties of squares:

- $\forall s,t$ *At*(Agent,s,t) $\wedge$ Breeze(t) $\Rightarrow$ Breezy(s)

Squares are breezy near a pit:

- Diagnostic rule---infer cause from effect
  $\forall s$ Breezy(s) $\Rightarrow \exists$ r, Adjacent(r,s) $\wedge$ Pit(r)
- Causal rule---infer effect from cause
  $\forall r$ Pit(r) $\Rightarrow$ [$\forall s$ Adjacent(r,s) $\Rightarrow$ Breezy(s)]

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: better to define wumpus world