CS 4410, Fall 2018 – Automata and Formal Languages
Final Exam
Due December 12, 2018 by 1:15 pm

- This is a take-home exam. All answers must be your own work. You may:
  - Discuss this exam with me,
  - Use our text book, class notes and handouts,
  - **No other sources are allowed.**
- To receive full credit, **show your work** and **write legibly**.
- If you need clarification about any of the problems, please ask me.

Name: _____

1. Let language $L = \{w \in \{0,1\}^* \mid$ w begins and ends with the same symbol, and the second symbol in w is the same as its second-to-last symbol$\}$. For example, 01101110 $\in L$ and 101010 $\notin L$.

   a. Give a regular expression that denotes L.

   b. Give an NFA (transition graph) that accepts L.

2. Let L_1 and L_2 be regular languages. Carefully prove that $L_1 \cap L_2$ is a regular language. (Hint: DeMorgan's law)

3. Let $L = \{ a^n b^j \mid n \leq j \}$, prove that L is not regular.

4. Let $L = \{ ww \mid w \in \{a, b\}^* \}$, prove that L is not regular.

5. Find a context-free grammar for $L = \{ a^n b^m c^k \mid k = n + m, \ where \ m \geq 0, n \geq 0 \}$.

   a. Find a context-free grammar for L.

   b. Construct an NPDA that accepts L.

6. Explain what it means for a problem to be undecidable.

7. Give an example of a problem that is NP hard.