

Math 2300, Spring 2017

Proofs using the Principle of **Strong** Mathematical Induction

Principle of Strong Mathematical Induction (Epp, p. 268)

Let $P(n)$ be a property that is defined for integers n , and let a and b be fixed integers with $a \leq b$. Suppose the following two statements are true:

1. $P(a), P(a + 1), \dots$, and $P(b)$ are all true. (basis step)
2. For any integer $k \geq a$, if $P(i)$ is true for all integers i from a through k , then $P(k + 1)$ is true. (inductive step)

Then the statement
for all integers $n \geq a$, $P(n)$ is true.

(The supposition that $P(i)$ is true for all integers i from a through k is called the inductive hypothesis. Another way to state the inductive hypothesis is to say that $P(a), P(a + 1), \dots, P(k)$ are all true.)

Dr. Martin's 5 step method to prove $P(n)$ for all integers $n \geq a$.

1. Base Cases: $P(a), P(a + 1), \dots, P(b)$

If it is an equation be sure to check both sides separately.

2. Write down the Induction Hypothesis: $n=i$

Where k is a particular, but arbitrary integer greater than or equal to b .

Let $k \in \mathbb{Z} \ni k \geq b$ and suppose $P(i) \forall i \in \mathbb{Z} \ni a \leq i \leq k$

3. Write down what needs to be proved: $n = k + 1$

We must show that $P(k+1)$

4. Show $P(a)$ through $P(k)$ implies $P(k+1)$ by clear and convincing argument, usually starts:

Consider $P(k+1)$

5. Conclusion

By PSMI $P(n) \forall n \in \mathbb{Z} \ni n \geq a$