CS 4410, Fall 2017 – Automata and Formal Languages

Final Exam

Due December 18, 2017 by 10:30 am

* This is a take-home exam. All answers must be your own work. You may:
	+ Discuss this exam with me,
	+ Use our text book, class notes and handouts,
	+ **No other sources are allowed.**
* To receive full credit, **show your work** and **write legibly**.
* If you need clarification about any of the problems, please ask me.

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Let language $L=\{w\in \left\{0,1\right\}^{\*} | $w begins and ends with the same symbol, and the second

symbol in w is the same as its second-to-last symbol}. For example, 01101110 $\in $ *L* and

101010 $\notin $ *L.*

 a. Give a regular expression that denotes *L* or explain why this cannot be done.

 b. Give an NFA (transition graph) that accepts *L* or explain why this cannot be done.

2. Let *L1* and *L2* be regular languages. Carefully prove that *L1* $∩$ *L2* is a regular language.

(Hint: DeMorgan's law)

3. Let $L=\left\{ a^{n}b^{j}a^{k} \right| n=j or j\ne k \}$, prove that *L* is not regular.

4. Let $L=\left\{ a^{n} \right| n= k^{2} for some k \geq 0 \}$, prove that *L* is not regular.

5. Prove or provide a counter example to the statement:

If *L1* and *L2* are non-regular languages, then *L1* $∪$ *L2* is a non-regular language.

6. Find a context-free grammar for $L=\left\{ a^{n}b^{m}c^{k} \right| k=n+m, where m\geq 0, n\geq 0 \}$.

 a. Find a context-free grammar for *L*.

 b. Construct an NPDA that accepts $L.$

7. What language is accepted by the following NPDA? M = ({q0, q1, q2}, {a, b}, {a, b, z}, q0, z, {q2}) with transitions

(q0, a, z) = {(q1, a), (q2, $λ$)}

(q1, b, a) = {(q1, b)}

(q1, b, b) = {(q1, b)}

(q1, a, b) = {(q2, $λ$)}

8. Is the following language context‐free? Justify your answer. If it is CFL, give a grammar.

Otherwise prove that the language is not CF using the pumping lemma.

$$L=\left\{ a^{n}b^{m} \right| n\ne m \}$$