

choose constant c so that:

$$T(m,2) \leq cm$$

$$T(2,n) \leq cn$$

$$T(m,n) \leq cmn + T(q, n/2) + T(m-q, n/2) \quad (\text{for } 1 \leq q \leq m)$$

Claim: $T(m,n) \leq 2cmn$ for all $0 \leq m, n$

Base cases:

$$T(m,2) \leq cm \leq 4cm = (2c)(m)(2) = 2cmn$$

$$T(2,n) \leq cn \leq 4cn = (2c)(2)(n) = 2cmn$$

Inductive Hypothesis:

$$T(m',n') \leq 2cm'n' \text{ for all } m' + n' < m + n$$

$$\begin{aligned} T(m,n) &\leq T(q, n/2) + T(m-q, n/2) + cmn \\ &\leq 2cq(n/2) + 2c(m-q)(n/2) + cmn \\ &= cqn + cmn - cq(n/2) + cmn \\ &= 2cmn \end{aligned}$$