

## 5. DIVIDE AND CONQUER I

, merge and count demo

Lecture slides by Kevin Wayne
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## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B


## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 2 and add $x$ to inversion count
sorted list C
$\uparrow$

$$
\begin{aligned}
& x=5 \\
& \text { inversions }=0
\end{aligned}
$$

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

| 3 | 7 | 10 | 14 | 18 | 2 | 11 | 16 | 17 | 23 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 |  |  |  |  | 5 | 4 |  |  |  |

compare minimum entry in each list: copy 3 and decrement $x$
sorted list C
2
$\uparrow$

$$
\begin{aligned}
& x=5 \\
& \text { inversions }=5
\end{aligned}
$$

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 7 and decrement $x$
sorted list C
23
$\uparrow$

$$
\begin{aligned}
& x=4 \\
& \text { inversions = } 5
\end{aligned}
$$

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 10 and decrement $x$
sorted list C
$\begin{array}{lll}2 & 3 & 7\end{array}$
4

$$
\begin{aligned}
& x=3 \\
& \text { inversions }=5
\end{aligned}
$$

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 11 and add $x$ to increment count
sorted list C
$\begin{array}{llll}2 & 3 & 7 & 10\end{array}$

\[

\]

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 14 and decrement $x$
sorted list C
$\begin{array}{lllll}2 & 3 & 7 & 10 & 11\end{array}$

$$
\begin{aligned}
& \text { 4 } \\
& x=2 \\
& \text { inversions }=7
\end{aligned}
$$

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 16 and add $x$ to increment count
sorted list C

| 2 | 3 | 7 | 10 | 11 | 14 |
| :--- | :--- | :--- | :--- | :--- | :--- |

$\uparrow$
$x=1$
inversions = 7

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 17 and add $x$ to increment count
sorted list C

$$
\begin{array}{lllllll}
2 & 3 & 7 & 10 & 11 & 14 & 16
\end{array}
$$

```
x = 1
inversions = 8
```


## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A
sorted list B

compare minimum entry in each list: copy 18 and decrement $x$
sorted list C
$\begin{array}{llllllll}2 & 3 & 7 & 10 & 11 & 14 & 16 & 17\end{array}$

```
x = 1
inversions = 9
```


## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A sorted list B

list A exhausted: copy 23
sorted list C
$\begin{array}{lllllllll}2 & 3 & 7 & 10 & 11 & 14 & 16 & 17 & 18\end{array}$

$$
\begin{aligned}
& x=0 \\
& \text { inversions }=9
\end{aligned}
$$

## Merge and count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.
sorted list A sorted list B

done: return 9 inversions
sorted list C

| 2 | 3 | 7 | 10 | 11 | 14 | 16 | 17 | 18 | 23 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$$
\begin{aligned}
& x=0 \\
& \text { inversions }=9
\end{aligned}
$$

