

The now familiar back-substitution is:

$$\begin{aligned}
 cN^k &= T(N) - \lambda T(N/b) \\
 &= [DN^k + KN^k \log_b(N)] \\
 &\quad - \lambda [D(N/b)^k + K(N/b)^k \log_b(N/b)] \\
 &= DN^k [1 - (\lambda/b^k)] \\
 &\quad + KN^k [\log_b(N) - (\lambda/b^k) \{\log_b(N) - \log_b(b)\}] \\
 &= 0 + KN^k [\log_b(N) - 1 \{\log_b(N) - 1\}] = KN^k
 \end{aligned}$$

Thus, the constant $K=c$: the original $c > 0$ of the problem statement. Therefore $cN^k \log_b(N)$ is the dominant term in $T(N) = DN^k + KN^k \log_b(N)$, so

$T(N)$ is $\Theta(N^k \log_b(N))$ when $\lambda = b^k$.

Summary: (c.f. page 135 of Brassard & Bratley)

when we have $T(N) = \lambda T(N/b) + cN^k$, $\forall N > N_0$;

$\lambda \in \mathbb{Z}$, $b \in \mathbb{Z}$; $\lambda \geq 1$, $b \geq 1$, $c \in \mathbb{R}$, $k \in \mathbb{R}$; $c \geq 0$, $k \geq 0$, and

$T: \mathbb{Z}^+ \rightarrow \mathbb{R}^+$ non-decreasing, then

$$T(N) = AN^{\log_b(\lambda)} + BN^k \text{ is } \Theta(N^k) \quad \text{when } \lambda < b^k$$

$$T(N) = DN^k + KN^k \log_b(N) \text{ is } \Theta(N^k \log_b(N)) \quad \text{when } \lambda = b^k$$

$$T(N) = AN^{\log_b(\lambda)} + BN^k \text{ is } \Theta(N^{\log_b(\lambda)}) \quad \text{when } \lambda > b^k$$