

An interesting exercise in wavelet theory is to find subsets of \mathbb{R} that tile the real line via integer translations and also successive dilations by a fixed dilation factor. Such a set is called a *wavelet set*. For instance, the interval $A = [0, 1]$ tiles the real line via integer translations since $A + 1 = [1, 2]$, $A + 2 = [2, 3]$, and so on with $A + n = [n, n + 1]$ for any $n \in \mathbb{Z}$. (Note that by the term *tile* we mean to partition the real line with the only gaps/overlaps being single points.) Furthermore, the set $B = [-2, -1] \cup [1, 2]$ tiles the real line via dilations by the factor of 2 since $2^n B = [-2^{n+1}, -2^n] \cup [2^n, 2^{n+1}]$. However, neither of these sets tile \mathbb{R} by both translations *and* dilations. In a joint paper with Marcin Bownik of the University of Oregon, we have characterized all possible sets consisting of either two or three intervals that fit this criteria. (It is easy to see that that exist no wavelet sets consisting of only one interval.) The mathematics involved is accessible to all students as it requires nothing more than solving systems of linear equations.