logically equivalent. On the other hand, truth tables can always be used to determine both equivalence and nonequivalence, and truth tables are easy to program on a computer. When truth tables are used, however, checking for equivalence always requires 2^n steps, where n is the number of variables. Sometimes you can quickly see that two statement forms are equivalent by Theorem 1.1.1, whereas it would take quite a bit of calculating to show their equivalence using truth tables. For instance, it follows immediately from the associative law for $\wedge$ that $p \wedge (\sim q \wedge \sim r) \equiv (p \wedge \sim q) \wedge \sim r$, whereas a truth table verification requires constructing a table with eight rows.

Exercise Set 1.1

Appendix B contains either full or partial solutions to all exercises with blue numbers. When the solution is not complete, the exercise number has an ***H*** next to it. A ∗ next to an exercise number signals that the exercise is more challenging than usual. Be careful not to get into the habit of turning to the solutions too quickly. Make every effort to work exercises on your own before checking your answers. See the Preface for additional sources of assistance and further study.

In each of 1–4 represent the common form of each argument using letters to stand for component sentences, and fill in the blanks so that the argument in part (b) has the same logical form as the argument in part (a).

1. a. If all integers are rational, then the number 1 is rational.
All integers are rational.
Therefore, the number 1 is rational.

b. If all algebraic expressions can be written in prefix notation, then ________________.
________________.
Therefore, $(a + 2b)(a^2 - b)$ can be written in prefix notation.

2. a. If all computer programs contain errors, then this program contains an error.
This program does not contain an error.
Therefore, it is not the case that all computer programs contain errors.

b. If ______, then ______.
2 is not odd.
Therefore, it is not the case that all prime numbers are odd.

3. a. This number is even or this number is odd.
This number is not even.
Therefore, this number is odd.

b. ______ or logic is confusing.
My mind is not shot.
Therefore, ______.

4. a. If n is divisible by 6, then n is divisible by 3.
If n is divisible by 3, then the sum of the digits of n is divisible by 3.
Therefore, if n is divisible by 6, then the sum of the digits of n is divisible by 3.
(Assume that n is a particular, fixed integer.)

b. If ________________,
then the guard condition for the **while** loop is false.
If ________________,
then program execution moves to the next instruction following the loop.
Therefore, if x equals 0, then ________________.
(Assume that x is a particular variable in a particular computer program.)

5. Indicate which of the following sentences are statements.

a. 1,024 is the smallest four-digit number that is a perfect square.

b. She is a mathematics major.

c. $128 = 2^6$ d. $x = 2^6$

Write the statements in 6–9 in symbolic form using the symbols $\sim$, $\vee$, and $\wedge$ and the indicated letters to represent component statements.

6. Let s = "stocks are increasing" and i = "interest rates are steady."

a. Stocks are increasing but interest rates are steady.

b. Neither are stocks increasing nor are interest rates steady.

7. Juan is a math major but not a computer science major. (m = "Juan is a math major," c = "Juan is a computer science major")

8. Let h = "John is healthy," w = "John is wealthy," and s = "John is wise."

a. John is healthy and wealthy but not wise.

b. John is not wealthy but he is healthy and wise.

c. John is neither healthy, wealthy, nor wise.

d. John is neither wealthy nor wise, but he is healthy.

e. John is wealthy, but he is not both healthy and wise.

9. Either Olga will go out for tennis or she will go out for track but not both. (n = "Olga will go out for tennis," k = "Olga will go out for track")

10. Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentences in symbolic notation.

a. DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.

b. DATAENDFLAG is off but ERROR is not equal to 0.

c. DATAENDFLAG is off; however ERROR is not 0 or SUM is greater than or equal to 1,000.

d. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.
e. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

11. In the following sentence, is the word *or* used in its inclusive or exclusive sense? A team wins the playoffs if it wins two games in a row or a total of three games.

In 12 and 13, imagine that you are searching the Internet using a search engine that uses AND for *and*, NOT for *not*, and OR for *or.*

12. You are trying to find the name of the fourteenth president of the United States of America. Write a logical expression to find Web pages containing the following: "United States president" and either "14th" or "fourteenth" but not "amendment" (to avoid pages about the Fourteenth Amendment to the United States Constitution).

13. You recall that the fastest mammal on earth is either a jaguar or a cheetah. To find a Web page to tell you which one is the fastest, write a logical expression containing "jaguar" and "cheetah,"and either "speed" or "fastest" but not "car," or "automobile," or "auto" (to avoid pages about the Jaguar automobile).

Write truth tables for the statement forms in 14–18.

14. $\sim p \wedge q$

15. $\sim(p \wedge q) \vee (p \vee q)$

16. $p \wedge (q \wedge r)$

17. $p \wedge (\sim q \vee r)$

***H* 18.** $(p \vee (\sim p \vee q)) \wedge \sim(q \wedge \sim r)$

Determine which of the pairs of statement forms in 19–28 are logically equivalent. Justify your answers using truth tables and include a few words of explanation. Read **t** to be a tautology and **c** to be a contradiction.

19. $p \vee (p \wedge q)$ and p

20. $\sim(p \wedge q)$ and $\sim p \wedge \sim q$

21. $p \vee \mathbf{t}$ and $\mathbf{t}$

22. $p \wedge \mathbf{t}$ and p

23. $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$

24. $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

25. $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$

26. $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

27. $((\sim p \vee q) \wedge (p \vee \sim r)) \wedge (\sim p \vee \sim q)$ and $\sim(p \vee r)$

28. $(r \vee p) \wedge ((\sim r \vee (p \wedge q)) \wedge (r \vee q))$ and $p \wedge q$

Use De Morgan's laws to write negations for the statements in 29–34.

29. Hal is a math major and Hal's sister is a computer science major.

30. Sam is an orange belt and Kate is a red belt.

31. The connector is loose or the machine is unplugged.

32. This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.

33. The dollar is at an all-time high and the stock market is at a record low.

34. The train is late or my watch is fast.

Assume x is a particular real number and use De Morgan's laws to write negations for the statements in 35–38.

35. $-2 < x < 7$

36. $-10 < x < 2$

37. $1 > x \geq -3$

38. $0 > x \geq -7$

In 39 and 40, imagine that *num_orders* and *num_instock* are particular values, such as might occur during execution of a computer program. Write negations for the following statements.

39. (*num_orders* > 100 and *num_instock* ≤ 500) or *num_instock* < 200

40. (*num_orders* < 50 and *num_instock* > 300) or (50 ≤ *num_orders* < 75 and *num_instock* > 500)

Use truth tables to establish which of the statement forms in 41–44 are tautologies and which are contradictions.

41. $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

42. $(p \wedge \sim q) \wedge (\sim p \vee q)$

43. $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

44. $(\sim p \vee q) \vee (p \wedge \sim q)$

In 45 and 46 below, a logical equivalence is derived from Theorem 1.1.1. Supply a reason for each step.

45. $(p \wedge \sim q) \vee (p \wedge q) \equiv p \wedge (\sim q \vee q)$ by (a)
$\equiv p \wedge (q \vee \sim q)$ by (b)
$\equiv p \wedge \mathbf{t}$ by (c)
$\equiv p$ by (d)

Therefore, $(p \wedge \sim q) \vee (p \wedge q) \equiv p$.

46. $(p \vee \sim q) \wedge (\sim p \vee \sim q)$
$\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$ by (a)
$\equiv \sim q \vee (p \wedge \sim p)$ by (b)
$\equiv \sim q \vee \mathbf{c}$ by (c)
$\equiv \sim q$ by (d)

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$.

Use Theorem 1.1.1 to verify the logical equivalences in 47–51. Supply a reason for each step.

47. $(p \wedge \sim q) \vee p \equiv p$

48. $p \wedge (\sim q \vee p) \equiv p$

49. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$

50. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

51. $(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q) \equiv p$

approach—they emphasize the punishment). Yet the parents who promise the reward intend to suggest the punishment as well, and those who threaten the punishment will certainly give the reward if it is earned. Both sets of parents expect that their conditional statements will be interpreted as biconditionals.

Since we often (correctly) interpret conditional statements as biconditionals, it is not surprising that we may come to believe (mistakenly) that conditional statements are always logically equivalent to their inverses and converses. In formal settings, however, statements must have unambiguous interpretations. If-then statements can't sometimes mean "if-then" and other times mean "if and only if." When using language in mathematics, science, or other situations where precision is important, it is essential to interpret if-then statements according to the formal definition and not to confuse them with their converses and inverses.

Exercise Set 1.2

Rewrite the statements in 1–4 in if-then form.

1. This loop will repeat exactly N times if it does not contain a **stop** or a **go to.**

2. I am on time for work if I catch the 8:05 bus.

3. Freeze or I'll shoot.

4. Fix my ceiling or I won't pay my rent.

Construct truth tables for the statement forms in 5–11.

5. $\sim p \vee q \rightarrow \sim q$

6. $(p \vee q) \vee (\sim p \wedge q) \rightarrow q$

7. $p \wedge \sim q \rightarrow r$

8. $\sim p \vee q \rightarrow r$

9. $p \wedge \sim r \leftrightarrow q \vee r$

10. $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$

11. $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

12. Use the logical equivalence established in Example 1.2.3, $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$, to rewrite the following statement. (Assume that x represents a fixed real number.)

If $x > 2$ or $x < -2$, then $x^2 > 4$.

13. Use truth tables to verify the following logical equivalences. Include a few words of explanation with your answers.
a. $p \rightarrow q \equiv \sim p \vee q$ b. $\sim(p \rightarrow q) \equiv p \wedge \sim q$.

H 14. **a.** Show that the following statement forms are all logically equivalent.

$p \rightarrow q \vee r$, $p \wedge \sim q \rightarrow r$, and $p \wedge \sim r \rightarrow q$

b. Use the logical equivalences established in part (a) to rewrite the following sentence in two different ways. (Assume that n represents a fixed integer.)

If n is prime, then n is odd or n is 2.

15. Determine whether the following statement forms are logically equivalent:

$p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$

In 16 and 17, write each of the two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words of explanation.

16. If you paid full price, you didn't buy it at Crown Books. You didn't buy it at Crown Books or you paid full price.

17. If Rob is goalkeeper and Aaron plays forward, then Sam plays defense. Rob is not goalkeeper or Aaron does not play forward or Sam plays defense.

18. Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Include truth tables and a few words of explanation.

If it walks like a duck and it talks like a duck, then it is a duck.

Either it does not walk like a duck or it does not talk like a duck, or it is a duck.

If it does not walk like a duck and it does not talk like a duck, then it is not a duck.

19. True or false? The negation of "If Sue is Luiz's mother, then Deana is his cousin" is "If Sue is Luiz's mother, then Deana is not his cousin."

20. Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)
a. If P is a square, then P is a rectangle.
b. If today is New Year's Eve, then tomorrow is January.
c. If the decimal expansion of r is terminating, then r is rational.
d. If n is prime, then n is odd or n is 2.
e. If x is nonnegative, then x is positive or x is 0.
f. If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
g. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

21. Suppose that p and q are statements so that $p \to q$ is false. Find the truth values of each of the following:
    a. $\sim p \to q$ b. $p \vee q$ c. $q \to p$

***H* 22.** Write contrapositives for the statements of exercise 20.

***H* 23.** Write the converse and inverse for each statement of exercise 20.

Use truth tables to establish the truth of each statement in 24–27.

24. A conditional statement is not logically equivalent to its converse.

25. A conditional statement is not logically equivalent to its inverse.

26. A conditional statement and its contrapositive are logically equivalent to each other.

27. The converse and inverse of a conditional statement are logically equivalent to each other.

***H* 28.** "Do you mean that you think you can find out the answer to it?" said the March Hare.

"Exactly so," said Alice.

"Then you should say what you mean," the March Hare went on.

"I do," Alice hastily replied; "at least—at least I mean what I say—that's the same thing, you know."

"Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

— from "A Mad Tea-Party" in *Alice in Wonderland,* by Lewis Carroll

The Hatter is right. "I say what I mean" is not the same thing as "I mean what I say." Rewrite each of these two sentences in if-then form and explain the logical relation between them. (This exercise is referred to in the introduction to Chapter 3.)

If statement forms P and Q are logically equivalent, then $P \leftrightarrow Q$ is a tautology. Conversely, if $P \leftrightarrow Q$ is a tautology, then P and Q are logically equivalent. Use $\leftrightarrow$ to convert each of the logical equivalences in 29–31 to a tautology. Then use a truth table to verify each tautology.

29. $p \to (q \vee r) \equiv (p \wedge \sim q) \to r$

30. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

31. $p \to (q \to r) \equiv (p \wedge q) \to r$

Use the contrapositive to rewrite the statements in 32 and 33 in if-then form in two ways. Assume that *only if* has its formal, logical meaning.

32. The Cubs will win the pennant only if they win tomorrow's game.

33. Sam will be allowed on Signe's racing boat only if he is an expert sailor.

34. Taking the long view on your education, you go to the Prestige Corporation and ask what you should do in college to be hired when you graduate. The personnel director replies that you will be hired *only if* you major in mathematics or computer science, get a B average or better, and take accounting. You do, in fact, become a math major, get a B^+ average, and take accounting. You return to Prestige Corporation, make a formal application, and are turned down. Did the personnel director lie to you?

Some programming languages use statements of the form "r unless s." This means that as long as s does not happen, then r will happen. More formally,

> **Definition:** If r and s are statements,
>
> **r unless s** means if $\sim s$ then r.

In 35–37, rewrite the statements in if-then form.

35. Payment will be made on the fifth unless a new hearing is granted.

36. Ann will go unless it rains.

37. This door will not open unless a security code is entered.

In 38–41 (a) use the logical equivalences $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the given statement forms without using the symbol $\to$ or $\leftrightarrow$, and (b) use the logical equivalence $p \vee q \equiv \sim(\sim p \wedge \sim q)$ to rewrite each statement form using only $\wedge$ and $\sim$.

38. $p \wedge \sim q \to r$

39. $p \vee \sim q \to r \vee q$

40. $(p \to r) \leftrightarrow (q \to r)$

41. $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

42. Given any statement form, is it possible to find a logically equivalent form that uses only $\sim$ and $\wedge$? Justify your answer.

Rewrite the statements in 43 and 44 in if-then form.

43. Catching the 8:05 bus is a sufficient condition for my being on time for work.

44. Having two 45° angles is a sufficient condition for this triangle to be a right triangle.

Use the contrapositive to rewrite the statements in 45 and 46 in if-then form in two ways.

45. Being divisible by 3 is a necessary condition for this number to be divisible by 9.

46. Doing homework regularly is a necessary condition for Jim to pass the course.

Note that "a sufficient condition for s is r" means r is a sufficient condition for s and that "a necessary condition for s is r" means r is a necessary condition for s. Rewrite the statements in 47 and 48 in if-then form.

Exercise Set 1.3

Use modus ponens or modus tollens to fill in the blanks in the arguments of 1–5 so as to produce valid inferences.

1. If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers a and b.
It is not true that $\sqrt{2} = a/b$ for some integers a and b.
$\therefore$ ____________________.

2. If this is a **while** loop, then the body of the loop may never be executed.
____________________.
$\therefore$ The body of the loop may never be executed.

3. If logic is easy, then I am a monkey's uncle.
I am not a monkey's uncle.
$\therefore$ ____________________.

4. If this figure is a quadrilateral, then the sum of its interior angles is 360°.
The sum of the interior angles of this figure is not 360°.
$\therefore$ ____________________.

5. If they were unsure of the address, then they would have telephoned.
____________________.
$\therefore$ They were sure of the address.

Use truth tables to determine whether the argument forms in 6–11 are valid. Indicate which columns represent the premises and which represent the conclusion, and include a few words of explanation to support your answers.

6. $p \to q$
$q \to p$
$\therefore p \lor q$

7. p
$p \to q$
$\sim q \lor r$
$\therefore r$

8. $p \lor q$
$p \to \sim q$
$p \to r$
$\therefore r$

9. $p \land q \to \sim r$
$p \lor \sim q$
$\sim q \to p$
$\therefore \sim r$

10. $p \to r$
$q \to r$
$\therefore p \lor q \to r$

11. $p \to q \lor r$
$\sim q \lor \sim r$
$\therefore \sim p \lor \sim r$

12. Use a truth table to prove the validity of modus tollens.

$$p \to q$$
$$\sim q$$
$$\therefore \sim p$$

13. Use truth tables to show that the following forms of argument are invalid.

a. $p \to q$
q
$\therefore p$
(converse error)

b. $p \to q$
$\sim p$
$\therefore \sim q$
(inverse error)

Use truth tables to show that the argument forms referred to in 14–21 are valid. Indicate which columns represent the premises and which represent the conclusion, and include a few words of explanation to support your answers.

14. Example 1.3.4(a)
15. Example 1.3.4(b)
16. Example 1.3.5(a)
17. Example 1.3.5(b)
18. Example 1.3.6(a)
19. Example 1.3.6(b)
20. Example 1.3.7
21. Example 1.3.8

Use symbols to write the logical form of each argument in 22 and 23, and then use a truth table to test the argument for validity. Indicate which columns represent the premises and which represent the conclusion, and include a few words of explanation to support your answers.

22. If Tom is not on team A, then Hua is on team B.
If Hua is not on team B, then Tom is on team A.
$\therefore$ Tom is not on team A or Hua is not on team B.

23. Oleg is a math major or Oleg is an economics major.
If Oleg is a math major, then Oleg is required to take Math 362.
$\therefore$ Oleg is an economics major or Oleg is not required to take Math 362.

Some of the arguments in 24–32 are valid, whereas others exhibit the converse or the inverse error. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

24. If Jules solved this problem correctly, then Jules obtained the answer 2.
Jules obtained the answer 2.
$\therefore$ Jules solved this problem correctly.

25. This real number is rational or it is irrational.
This real number is not rational.
$\therefore$ This real number is irrational.

26. If I go to the movies, I won't finish my homework.
If I don't finish my homework, I won't do well on the exam tomorrow.
$\therefore$ If I go to the movies, I won't do well on the exam tomorrow.

27. If this number is larger than 2, then its square is larger than 4.
This number is not larger than 2.
$\therefore$ The square of this number is not larger than 4.

28. If there are as many rational numbers as there are irrational numbers, then the set of all irrational numbers is infinite.
The set of all irrational numbers is infinite.
$\therefore$ There are as many rational numbers as there are irrational numbers.

29. If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.
Neither of these two numbers is divisible by 6.
∴ The product of these two numbers is not divisible by 6.

30. If this computer program is correct, then it produces the correct output when run with the test data my teacher gave me.
This computer program produces the correct output when run with the test data my teacher gave me.
∴ This computer program is correct.

31. Sandra knows Java and Sandra knows C++.
∴ Sandra knows C++.

32. If I get a Christmas bonus, I'll buy a stereo.
If I sell my motorcycle, I'll buy a stereo.
∴ If I get a Christmas bonus or I sell my motorcycle, then I'll buy a stereo.

33. Give an example (other than Example 1.3.13) of a valid argument with a false conclusion.

34. Give an example (other than Example 1.3.14) of an invalid argument with a true conclusion.

35. Explain in your own words what distinguishes a valid form of argument from an invalid one.

36. Given the following information about a computer program, find the mistake in the program.
a. There is an undeclared variable or there is a syntax error in the first five lines.
b. If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.
c. There is not a missing semicolon.
d. There is not a misspelled variable name.

37. In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a–e below) and challenged the reader to use them to figure out the location of the treasure.
a. If this house is next to a lake, then the treasure is not in the kitchen.
b. If the tree in the front yard is an elm, then the treasure is in the kitchen.
c. This house is next to a lake.
d. The tree in the front yard is an elm or the treasure is buried under the flagpole.
e. If the tree in the back yard is an oak, then the treasure is in the garage.
Where is the treasure hidden?

38. You are visiting the island described in Example 1.3.16 and have the following encounters with natives.
a. Two natives A and B address you as follows:
A says: Both of us are knights.
B says: A is a knave.
What are A and B?
b. Another two natives C and D approach you but only C speaks.
C says: Both of us are knaves.
What are C and D?
c. You then encounter natives E and F.
E says: F is a knave.
F says: E is a knave.
How many knaves are there?
H **d.** Finally, you meet a group of six natives, U, V, W, X, Y, and Z, who speak to you as follows:
U says: None of us is a knight.
V says: At least three of us are knights.
W says: At most three of us are knights.
X says: Exactly five of us are knights.
Y says: Exactly two of us are knights.
Z says: Exactly one of us is a knight.
Which are knights and which are knaves?

39. The famous detective Percule Hoirot was called in to solve a baffling murder mystery. He determined the following facts:
a. Lord Hazelton, the murdered man, was killed by a blow on the head with a brass candlestick.
b. Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder.
c. If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine.
d. If Lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.
e. If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.
f. If Sara was in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.
Is it possible for the detective to deduce the identity of the murderer from the above facts? If so, who did murder Lord Hazelton? (Assume there was only one cause of death.)

40. Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:
a. Socko: Lefty killed Sharky.
b. Fats: Muscles didn't kill Sharky.
c. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
d. Muscles: Lefty didn't kill Sharky.
Who did kill Sharky?

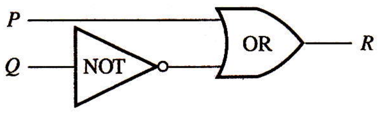
In 41–44 a set of premises and a conclusion are given. Use the valid argument forms listed in Table 1.3.1 to deduce the conclusion from the premises, giving a reason for each step as in Example 1.3.10. Assume all variables are statement variables.

$$
\begin{aligned}
\text{b. } P \vee Q &\equiv \sim(\sim(P \vee Q)) && \text{by the double negative law (Theorem 1.1.1(6))}\\
&\equiv \sim(\sim P \wedge \sim Q) && \text{by De Morgan's laws (Theorem 1.1.1(9))}\\
&\equiv \sim((P \mid P) \wedge (Q \mid Q)) && \text{by part (a)}\\
&\equiv (P \mid P) \mid (Q \mid Q) && \text{by definition of } \mid.
\end{aligned}
$$
■

Exercise Set 1.4

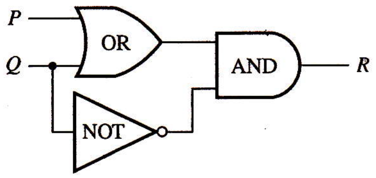
Give the output signals for the circuits in 1–4 if the input signals are as indicated.

1.



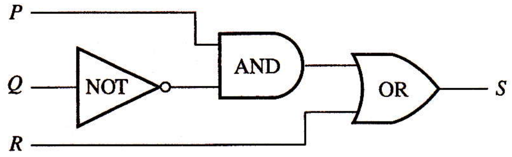
input signals: $P = 1$ and $Q = 1$

2.



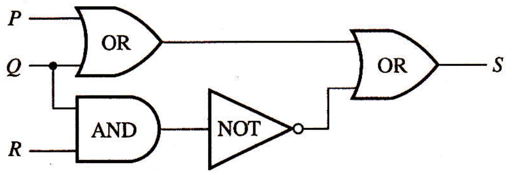
input signals: $P = 1$ and $Q = 0$

3.



input signals: $P = 1$, $Q = 0$, $R = 0$

4.



input signals: $P = 0$, $Q = 0$, $R = 0$

In 5–8, write an input/output table for the circuit in the referenced exercise.

5. Exercise 1 6. Exercise 2

7. Exercise 3 8. Exercise 4

In 9–12, find the Boolean expression that corresponds to the circuit in the referenced exercise.

9. Exercise 1 10. Exercise 2

11. Exercise 3 12. Exercise 4

Construct circuits for the Boolean expressions in 13–17.

13. $\sim P \vee Q$ 14. $\sim(P \vee Q)$

15. $P \vee (\sim P \wedge \sim Q)$ **16.** $(P \wedge Q) \vee \sim R$

17. $(P \wedge \sim Q) \vee (\sim P \wedge R)$

For each of the tables in 18–21, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table.

18.

P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

19.

P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	0

20.

P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

21.

P	*Q*	*R*	*S*
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	0

22. Design a circuit to take input signals P, Q, and R and output a 1 if, and only if, P and Q have the same value and Q and R have opposite values.

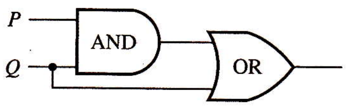
23. Design a circuit to take input signals P, Q, and R and output a 1 if, and only if, all three of P, Q, and R have the same value.

24. The lights in a classroom are controlled by two switches: one at the back and one at the front of the room. Moving either switch to the opposite position turns the lights off if they are on and on if they are off. Assume the lights have been installed so that when both switches are in the down position, the lights are off. Design a circuit to control the switches.

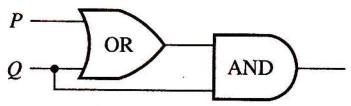
25. An alarm system has three different control panels in three different locations. To enable the system, switches in at least two of the panels must be in the on position. If fewer than two are in the on position, the system is disabled. Design a circuit to control the switches.

Use the properties listed in Theorem 1.1.1 to show that each pair of circuits in 26–29 have the same input/output table. (Find the Boolean expressions for the circuits and show that they are logically equivalent when regarded as statement forms.)

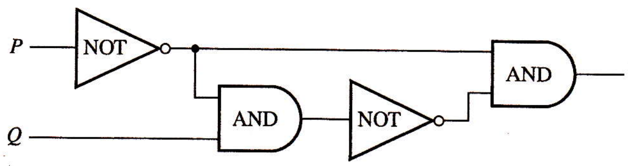
26. a.



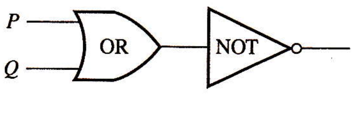
b.



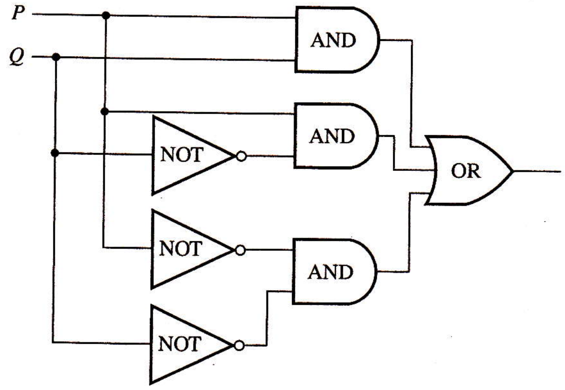
27. a.



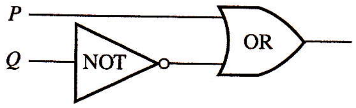
b.



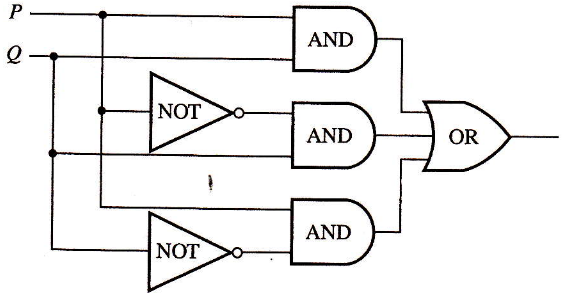
28. a.



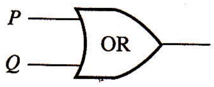
b.



29. a.



b.



For the circuits corresponding to the Boolean expressions in each of 30 and 31 there is an equivalent circuit with at most two logic gates. Find such a circuit.

30. $(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

31. $(\sim P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$

32. The Boolean expression for the circuit in Example 1.4.5 is

$$(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$$

(a disjunctive normal form). Find a circuit with at most three logic gates that is equivalent to this circuit.

33. a. Show that for the Sheffer stroke |,

$$P \wedge Q \equiv (P \mid Q) \mid (P \mid Q).$$

b. Use the results of Example 1.4.7 and part (a) above to write $P \wedge (\sim Q \vee R)$ using only Sheffer strokes.